AN ELEMENTARY COURSE OF HYDROSTATICS AND SOUND,

DESIGNED FOR

THE USE OF SCHOOLS, COLLEGES,

AND

CANDIDATES FOR UNIVERSITY AND OTHER EXAMINATIONS.

BY

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PREFACE.

THE study of the sciences of Mechanics and Hydrostatics is important both on account of the positive knowledge which may be acquired on the subjects belonging to these sciences, and also on account of the effects of the study as a discipline of the mind and an illustration of philosophical principles.

They have been termed *mixed* sciences, for they must be treated in part according to the inductive methods of physical science, and in part according to a rigorous system of mathematical reasoning.

In the present work on Hydrostatics, the two methods are taken separately. Part I., or the Inductive Part, treats chiefly of those principles of the science which are derived directly and professedly from experiment and observation.

In Part II., or the Deductive Part, other principles are deduced from the laws established in the

first part, from the definitions of terms, and from axioms or self-evident principles, by the same rigorous line of demonstration, and under the same logical laws, as the reasoning of Geometry.

Part III is devoted to Acoustics.

The whole contains all that is required on these subjects for the B.A. and B.Sc. degrees of the University of London.

The Metric System of Weights and Measures being the only system which furnishes an *exact* relation between the units of weight and units of capacity, has been extensively, though for obvious reasons not exclusively, used throughout the work.

CONTENTS.

HYDROSTATICS.

PART I.

I.—INTRODUCTION.

1. All matter with which we are acquainted presents itself in one of two forms, either *solid*, as iron, wood, stone, or *fluid*, as air, water, oil.

When a knife is used to separate into parts a block of wood, however sharp the knife may be, it will not enter the block unless a certain pressure be applied to it; but if a very thin plate of metal be placed in air or water, very little resistance will be experienced to its motion in the direction of its plane. This difference distinguishes the two classes of bodies. The particles of a solid body cohere, so that a force is necessary to separate them; the particles of a fluid may be separated by the slightest possible force. Hence a fluid differs from a solid by the absence of cohesion and friction. Although there is no fluid in nature between the particles of which there is an absolute absence of friction, nevertheless those substances which we shall consider as fluid fulfil this condition so nearly, as to make the

conclusions founded on it practically correct. Hence we shall define a *fluid as a collection of material particles which move freely amongst themselves, and consequently can be displaced by the slightest force.*

2. There are two classes of fluids—gases and liquids. The particles of the former not only yield to the smallest pressure, but have a repulsive force among themselves. There is no such repulsive force between the particles of a liquid. A cubic foot of any gas may readily be compressed into half a foot, double the pressure will reduce it to a quarter of a foot, and when the pressure is removed the gas returns to its original bulk; but no ordinary pressure produces any sensible compression on water or any other liquid. Hence gases have been denominated *elastic* fluids, and liquids *non-elastic.*

3. The perfect mobility of the particles of a fluid leads to special conditions of equilibrium. The science which treats of these conditions is termed Hydrostatics.

4. We have said that liquids cannot be compressed by any ordinary force; and this is so far true, that both in the theory of hydrostatics and in practice, they are assumed to be perfectly incompressible. But under great pressure liquids are slightly compressible. By sinking a vessel filled with fluid in the ocean to the depth of 2000 metres, where every square centimetre supports a weight of 200 kilogrammes, and every square inch nearly 3000 lbs., it is found that the volume of water is diminished one-twentieth.

5. If the pressure of the atmosphere be removed

from a given volume of water, its bulk increases by ·00005 of this volume. The increase in the case of mercury would be nearly ·000005 of its original bulk.

II.—THE TRANSMISSION OF PRESSURE.

6. In mechanics we have considered the action of forces on rigid or solid bodies. We have found, for example, that when two parallel forces are applied at the ends of a lever, there will be equilibrium if one particular point in the lever be fixed. This is a property of solid bodies. When, however, the forces act on fluids, the result is different. Each point in the fluid must be directly sustained.

Let us imagine a straight tube full of water, with a piston fitting one end. If a pressure of 1 lb. be exerted on the piston, the water will escape from the other end of the tube; to prevent this, a pressure of 1 lb. must be exerted in the opposite direction by means of an air-tight piston like the first. There is here nothing new: if instead of pistons and fluid we had a rigid rod of wood or metal, the same relation between the forces would produce equilibrium. But let us bend the tube in any way we please (Fig. 1), and the two equal pistons, A and B, placed at the two ends, will be in equilibrium if acted on perpendicularly to their surfaces by equal forces. It is not necessary that the forces should here be directly opposite. The pressure exerted by the piston A is

transmitted through the windings of the tube, and acts perpendicularly to the surface of the piston B, even when the direction is entirely different from

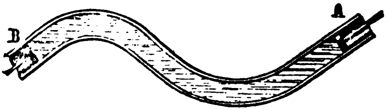

Fig. 1.

that of A. It is to this pressure at B that the force applied must be equal and opposite.

7. It must be remembered that we are speaking of the equilibrium of the fluid in the tube, and not of the tube itself. We suppose the tube to be fixed firmly on its support. The equilibrium of the solid tube and the equilibrium of the fluid are distinct things, and must be considered separately.

8. Take a closed vessel of water, and make in it an aperture B (Fig. 2), into which a piston with a surface of one square inch may be fitted, and exert a pressure of 1 lb. on this piston. We may imagine that a tube passes perpendicularly from the aperture B, traversing the mass of the liquid, and meeting the surface of the vessel again perpendicularly at a point A. Since there is equilibrium the fluid surrounding the portion A B must exert everywhere a pressure upon this portion equal to that which would be exerted by the surface marked off if it were that of a rigid tube. Hence, the pressure at B will be transmitted to A, and, in order that it may be in equilibrium, a force of 1 lb. must be applied externally.

As we may repeat the same hypothesis for each point in the surface of the vessel, we conclude that tho pressure exerted on B is transmitted by the liquid equally in every direction, so that every square

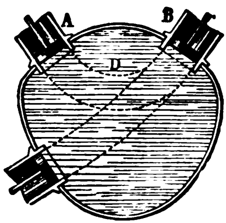

Fig. 2.

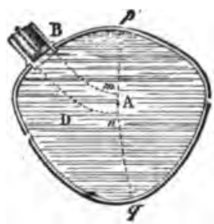

Fig. 3.

inch of surface is acted on by a normal pressure of 1 lb.

9. Imagine a surface *p q* (Fig. 3), within the liquid, and suppose a tube constructed from B to a point A in the surface, so as to cover at that point an area, *m n*, equal to one square inch; the surface *m n* will sustain a pressure of 1 lb. Moreover, since the mass of the liquid itself is in equilibrium, it follows that *m n* must also sustain a pressure of 1 lb. in the opposite direction. This important principle of fluids may be enunciated thus: *A pressure exerted at any point whatever in a fluid in equilibrium is transmitted undiminished to every point in the fluid.*

10. If there are two pistons, B and C, each equal to A, then when a pressure of 1 lb. is applied at A, a pressure of 1 lb. must also be applied at B and C. Suppose the pistons united so that they present the same area to the fluid, they will sustain the same

pressure—that is to say, twice the pressure on A. A piston having three times the area of A will sustain thrice the pressure, and generally the pressures sustained are proportional to the areas.

11. Suppose, for example, that the pistons are fitted to two tubes connected by a space filled with water (Fig. 4). Let the area of the smaller piston be one square unit, and that of the larger ten square units. When the smaller piston is pressed in with a force of 1 lb., it will be necessary to exert a pressure on the larger of 10 lbs., in order to prevent it from being moved. If a and b be the areas, and p and P the pressures on them, then $p : P :: a : b$ or $p\,b = a\,P$. If the radius diameter or circumference of one piston be n times the corresponding line of the other, the area of the first will be n^2 times the second, and the pressure on the first will therefore be n^2 times that on the second.*

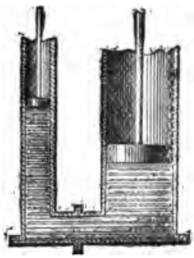

Fig. 4.

Example.—If the circumference of the smaller be 2 in., and that of the larger 15 in., the pressure p on the former is to the pressure P on the latter as $2^2 : 15^2$. Let the pressure on the former be 1 lb., then that on the latter is $15^2 \div 2^2$, or 56·25 lbs.

12. The principle of virtual velocities applies to this as to every other mechanical power. Suppose,

* It must be remembered that the *ratio* of the areas is what is here wanted, the area of a circle is the square of the radius multiplied by π (or approximately, by $\frac{22}{7}$).

for example, the small piston to decend n in., the amount of water driven out of the smaller cylinder into the larger will be $a \times n$ cubic in. If the larger piston ascend m in., the amount of water forced into it will be $b \times m$; hence

$$
\begin{array}{lrcl}
 & a \times n & = & b \times m \\
\text{but} & b \times p & = & a \times P \\
\text{hence} & p \times n & = & P \times m
\end{array}
$$

The principle explained above is applied in the Bramah or Hydraulic Press.

The Bramah Press.

13. *Description of the Press.*—The whole instrument is represented in Fig. 5, and a section of it in

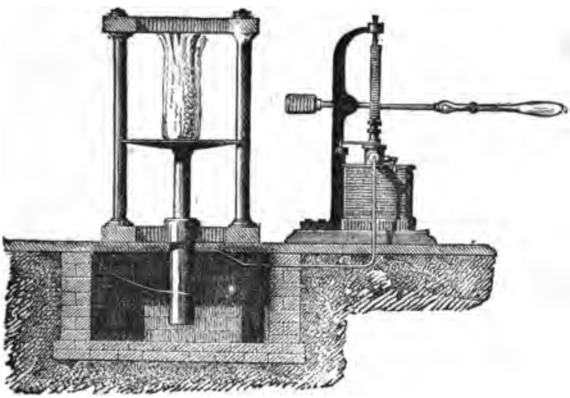

Fig. 5.

Fig. 6. a and b are two pistons working through air-tight collars in strong cylinders filled with water, and connected by a tube c c. The small piston a is

worked by a lever represented on the right of the upper figure, and the larger piston *b* is attached to a movable platform, on which the substance to be

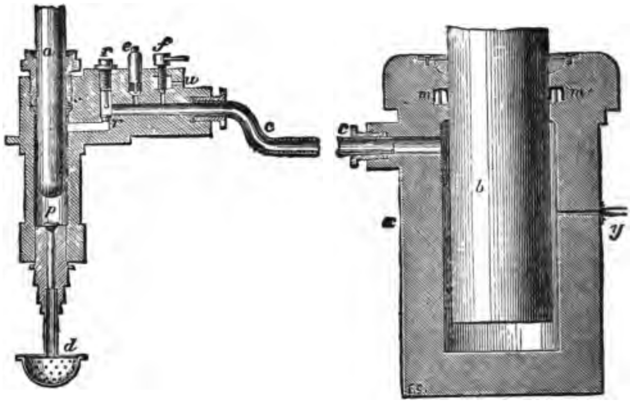

Fig. 6.

pressed is placed. At *h* and *r* are valves opening upwards. The tube *d* passes into a cistern of water.

Action of the Press.—Suppose the cylinder filled with water, and *a* in its lowest position; on raising *a*, the atmospheric pressure forces water from the reservoir through *d*, and when *a* is afterwards forced down, the valve *p* closes, the valve *r* is opened, a portion of the water is driven along *c*, and the cylinder *b* is made to ascend. By repeating this process any required compression of the substance between the platforms may be produced. At

f there is a plug, which can be unscrewed when the compression is completed.

14. The press is named from Mr. Bramah, who invented the simple and beautiful contrivance represented at *m m* in Fig. 6, for preventing the escape of the water from the cylinder. The ring or collar *m* is lined with leather, which is allowed to hang down like a flap in the cylinder. When water is forced upwards between the piston and the cylinder, it fills the fold of leather, and causes the edge to embrace the piston, so that the liquid cannot force its way higher than *m*. The greater the pressure, the more tightly will the flap close round the piston.

The relation between the power exerted on the press and the force at the end of the lever.

15. Let the diameter of the piston in the pump be 1·5 centimetres, and the diameter of the piston of the press 20 centimetres. Let the force be 18 kilogs., applied at 36 centimetres from the fulcrum, and let the piston of the pump be 6 centimetres from the fulcrum.

Force at a distance of 36 cents. from fulcrum = 18
" " 6 " " =108

$\therefore$ Pressure on area $(1{\cdot}5)^2 = 108$

" $1 = \frac{108}{2{\cdot}25} = 48$

" $20^2 = 48 \times 400 = 19{,}200$ kilogs.

16. In the same way we may find the relation for any dimensions.

Let *m* and *n* be the arms of the lever, F the force acting on the lever, P the pressure exerted by the

piston of the pump, a and b the areas of the pistons. Then, by the principle of the lever,

$$\text{the pressure on the smaller piston} = \mathrm{F}.\frac{m}{n}$$

$$\text{the pressure on one unit of area} = \mathrm{F}.\frac{m}{na}$$

$$\text{pressure on } b \text{ units of area} = \mathrm{F}\frac{mb}{na}$$

If R and r be the radii, $\quad \frac{b}{a} = \frac{\mathrm{R}^2}{r^2}$

$$\text{Hence the pressure on } b = \mathrm{F}\frac{m}{n}\frac{\mathrm{R}^2}{r^2}$$

Instead of a pressure of 1 lb. on the piston (Fig. 4), a pound of water in the tube above would produce the same effect. If the vertical tube holding this water be made twice as long, but of half the sectional area, the pressure on a unit of area will be doubled, while the same amount of fluid only will be used. By continually repeating this process, the small quantity of fluid may be made to support a very great weight. This principle, stated as follows, is sometimes termed the Hydrostatic Paradox:—*Any quantity of fluid, however small, may be made to support any weight, however large.*

Exercises on the Properties of Fluids.

1. Why does a liquid readily change its form when its position is changed?

2. What properties of liquids are not possessed by solids?

3. If a solid be placed inside a sphere, and a pressure be exerted upon the solid by a piston, how will the pressure be transmitted?

4. If the sphere be filled with a liquid, and pressure be exerted by means of the piston, how will the pressure act on the sphere?

5. If a vessel full of liquid be provided with two pistons, one of which is 20 square centimetres in area, and the other 100 square centimetres, and a pressure of 3 kilogs. be applied to the former, what pressure will be exerted on the latter?

$$\text{Pressure on } 20 \text{ square c.} = 3 \text{ kilogs.}$$
$$\text{,, on } 1 \text{ square c.} = \frac{3}{20}$$
$$\text{,, on } 100 \text{ square c.} = \frac{3 \times 100}{20} = 15.$$

6. If one piston be 10 centimetres in circumference, and the other 80, what must be the pressure applied to the smaller that the larger may support 1000 kilogs.?

$$\text{Pressure on area of } 80^2 = 1000$$
$$\text{,, ,, } 1 = \frac{1000}{6400}$$
$$\text{,, ,, } 10^2 = \frac{1000 \times 100}{6400} = 15\tfrac{5}{8}$$

7. If the area of the smaller piston be 12, and that of the larger 300, what pressure will the larger sustain when 5 kilogs. is applied to the smaller?

8. The diameter of the smaller piston being 3, and that of the larger 18, what pressure on the smaller will produce a pressure of 900 lbs. on the larger?—*Ans.* 25 lbs.

9. The circumferences of the pistons are as 4 to 55, and the pressure on the smaller 400 grammes, find that on the larger.—*Ans.* 75,625 grammes.

10. In descending 20 centimetres, the smaller piston raises a weight of 800 lbs. through a height of ·5 centimetre. What pressure is applied?—*Ans.* 20 lbs.

11. Two vertical tubes A and B are connected by a horizontal tube; A is short, and is provided with a piston; B is long, and has no piston. The diameter of A is 50 times that of B, what height of water in B will support 1000 kilogs. in the piston in A (a depth of a centimeter produces a pressure of one gramme on a square centimetre)?—*Ans.* 4 metres.

12. The pressure on a plane area in the form of a square, the side of which is a yard, is known to be uniform, and to be equal

to 2700 lbs.; find the pressure at a point when the unit of length is a sq. in.—*Ans.* $2\frac{1}{12}$ lbs.

13. The pressure at any point in a rectangular surface 1·5 metres long and 1·2 metres broad, is 100 grammes, a centimetre being the unit of length. Find the pressure on the whole surface. —*Ans.* 1800 kilogs.

14. If the diameter of the pump of a Bramah press be 3 centimetres, and that of the press 120 centimetres, what force will 1 kilog. applied to the pump produce on the press?—*Ans.* 1600 kilogs.

15. If a power of 100 kilogs. be applied to a lever at 30 centimetres from the fulcrum, the piston of the pump being attached 6 centimetres from the fulcrum, what will be the force acting on the press in the last question?—*Ans.* 800,000 kilogs.

III.—PRESSURE OF A FLUID ARISING FROM ITS WEIGHT.

17. Liquids have weight like solids; but the particles of a liquid glide over each other without friction. Hence, when a liquid is contained in a vessel, it is always found that the surface is in a horizontal plane. When the fluid is at rest, we see that this must be the case. If part of the fluid surface were inclined (Fig. 7), we should have particles acted on only by their weight at rest, on an inclined plane free from friction, which cannot be.

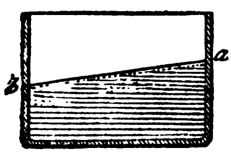

Fig. 7.

18. When the liquid is contained in several vessels communicating with each other, it is still found that the surfaces in the different vessels all lie in the same horizontal plane. Thus, if tubes of various shapes be connected as in the figure (Fig. 8), the water rises in all to the same level. This fact is frequently expressed by saying that liquids maintain

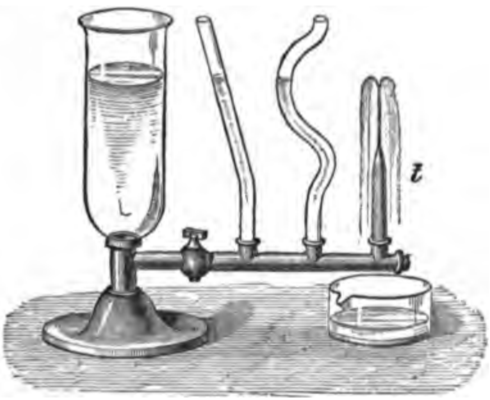

Fig. 8.

their level. If a small vertical tube *t* communicating with the vessels terminate below the level of the surface, and be pierced with a small hole at the top, a jet of water rises from the orifice to the level of the surface. The force with which the water leaves the aperture is proportional to the depth of the aperture below the surface.

19. This principle is applied in many ways. Water is distributed over a city by subterranean

pipes issuing from a large reservoir situated on an adjacent elevation. These pipes are terminated by stop-cocks, which are below the surface of the reservoir, so that when they are turned on the water escapes.

20. The law according to which the pressure produced by the weight of a fluid is transmitted in every direction frequently makes the effect of this weight very different from that of a solid. Suppose, for example, that a cylinder of ice is placed in a vessel of the same shape, the weight of the ice will exert a pressure on the base, but none on the sides of the vessel. The sides may be removed, while the mass within retains the same position. When, however the ice is melted, there is not only a pressure on the base equal to the weight, but also a pressure on the sides. If a hole be made in the vessel, the liquid will flow out, and the force with which it escapes will be proportional to the depth of the hole.

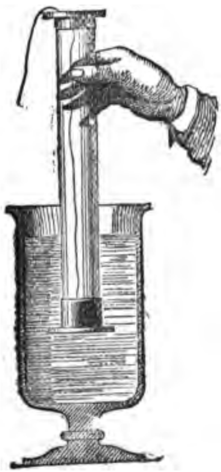

Fig. 9.

This variation in the pres sure may be illustrated by the following experiment.

21. Take a hollow cylinder or tube of glass, one end of which may be closed by a disc of metal supported by a thread (Fig. 9). Apply the disc to the tube,

and plunge the closed end in a vessel of water. At a certain depth, depending on the weight of the disc, the thread may be dropped, and the disc supported by the fluid pressure only. If the cylinder be moved horizontally, no change takes place. This result illustrates the fact that the pressure at points in the same horizontal plane are equal. If a heavier disc be taken, it must be plunged to a greater depth before it will be supported, and it will be found that the weight supported is proportional to the depth. Suppose, now, we take a very thin disc, make the cylinder descend to a certain depth, then pour water gently into the tube, we shall find that when the water in the tube reaches the level of that outside, the disc will be detached; being equally pressed on both sides, it falls by virtue of its weight. Hence the upward pressure on the disc is the weight of the liquid which would fill the cylinder to the level of the surrounding fluid.

22. Suppose the area of the disc and cylinder to be 40 square centimetres. The weight of a cubic centimetre is 1 gramme, and, with a base of 40 square centimetres, and height 1 centimetre, we get 40 cubic centimetres. Hence, at a depth of 1 centimetre, a disc weighing 40 grammes will be supported. At a depth of 2 centimetres the pressure of the fluid will be twice 40 grammes, and so on. Thus, for every square centimetre of area in the section of the cylinder there will be a pressure of as many grammes as there are centimetres in the depth of the disc below the surface.

23. The principle that the pressure depends on

the depth, and not on the quantity of fluid, may be further illustrated by the following series of experiments.* Take a balance having a circular disc of metal attached by a fine wire (Fig. 10) to one end of the beam, just heavy enough to balance the scale at the other. Take a cylindrical vessel open at both

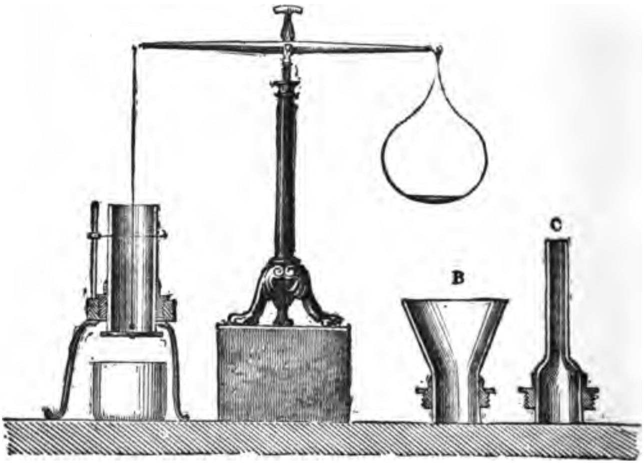

Fig. 10.

ends, and place it with axis vertical on a tripod, so that the wire supporting the disc passes along the axis of the cylinder. Let the disc and cylinder be properly ground, so that when pressed together they form a vessel which will hold water. Place a certain weight, a pound, for example, in the scale; the disc will now be pressed against the cylinder, and, if water be poured gently into the cylinder, the disc will remain

* Pascal's Vases.

in contact until the water rises to a certain height. If more water be added, the pressure will overcome the weight in the scale, and will force open the bottom. On a rod attached to the stand mark the height of the water when the highest level is reached. It will be found that exactly one pound of water will be supported. Pour the water out of the cylinder into the scale, taking away the 1 lb. weight, replace the cylinder, and fill it again gently up to the height h; the water in the scale will be just sufficient to keep the disc in contact with the cylinder. We see, therefore, that the pressure on the disc is equal to the weight of the column of water above it.

Repeat the experiment with vessels B and C (Fig. 10) of different shapes, having the area of the base the same. Water must be poured to the same height as before, in order that the pressure on the disc may be 1 lb. We shall find, however, that with vessel B there is more than 1 lb. of water, and with vessel C less than 1 lb. Thus the pressure on a plane does not depend on the amount of water used; it depends on the size of the plane and its depth below the surface. In all the cases the pressure is equal to the weight of a column of fluid having the plane for base and the depth of the plane for height. Only in the case of the cylinder is this the same as the weight of the water used. Thus we see that the pressure on the horizontal base of any vessel is the weight of the fluid which a vessel having the same base and vertical sides would contain, when filled to the same level.

Example.—What is the *pressure* on the base of a cone which has the vertex upwards and is filled with water to the depth of 18 centimetres, the radius of the base being 14 centimetres?

The area of the base is $14^2 \times \pi$ or $\frac{22}{7}$ of $14^2 =$ 616 square centimetres.

The volume of fluid which would be contained in a cylinder on the same base, and having the height of the water in the cone $= 616 \times 18 = 11{,}088$ cubic centimetres. Hence the pressure is 11,088 grammes.

The Pressure on the Sides of a Vessel.

24. The pressure on the sides of a vessel will not be uniform, but will be less near the top and greater near the bottom. Let us consider a vessel, the sides of which are rectangles. The pressure increases with the depth, and the mean pressure is that at half the depth. Hence the pressure on the rectangle would be equal to the weight of water above the rectangle, placed horizontally in a liquid at a depth equal to the depth of its middle point.

Example.—The rectangular side of a vessel is 50 centimetres long, and the water in contact with it is 20 centimetres deep. Find the pressure on the side.

The area equals $50 \times 20 = 1000$ square centimetres. The depth of the middle point is 10 C. Hence the pressure is equal to the weight of 1000×10, or 10,000 cubic centimetres of water; that is, 10,000 grammes.

25. If the surface be not a rectangle, the mean pressure will be that at the centre of gravity of the surface. Hence, if we suppose the side spread out

horizontally, and a column of the fluid resting on it to a height equal to the depth of the centre of gravity of the side, the weight of this column will be equal to the pressure on the side.

Example.—Find the pressure on a triangle whose base is 12, height 10, and centre of gravity at a depth of 30 centimetres.

Area $= \frac{1}{2} \times 12 \times 10 = 60.$

Pressure at depth of 1 cent. = 60 grammes.
" " 30 " = 1800 "

26. When embankments are made to streams or reservoirs they must increase in breadth and strength with the depth. The breadth of water makes no difference in the pressure when the water stands at the same level on both sides. The pressure on a dock-gate is the same on the inside, where the water is only a few yards across, as on the outside which faces the ocean.

27. The line of action of the resultant pressure on a surface will intersect the surface in a certain point, termed the centre of pressure. Suppose it is required to place a prop against the vertical side of a cistern, so as to give it additional support, at what height must the prop be applied? If the pressure on the rectangle forming the side were uniform, then the required point would be the centre, but the pressure varies as the depth, and is, therefore, greater on the lower half than on the upper, so that the prop must be applied at some point below the centre. To find the exact position of this point is to find the centre of pressure of the rectangle.

28. The centre of pressure P of a rectangle having its upper side in the surface of a fluid, is found to be in the line joining the middle points of its horizontal sides, at one-third from the base. Thus, if one side of the rectangle be under the action of the fluid pressure, and the other side be acted on only by a single force equal to the fluid pressure at the point P, the rectangle will remain at rest.

This is an important principle which has to be borne in mind in placing the hinges and supports of such surfaces as lock-gates.

Note.—In the following Exercises it is frequently necessary to connect units of volume with units of weight.

In the metric system, the principal unit of length is the *metre*. The unit, however, which is most frequently used in considering fluid pressures is the hundredth part of the metre, or the *centimetre*. The exact weight of a cubic centimetre of pure water at a certain definite temperature (4°) is one *gramme*. When we speak of the weight of a certain bulk of water, we shall always suppose the water to be pure, and to have a temperature of 4°. A kilogramme is 1000 grammes.

When English units are used, we cannot express *exactly* the relation between the volume of a cubic unit of water and its weight.

A cubic foot of water weighs *nearly* 1000 ounces.

A cubic inch ,, ,, $\frac{1000}{1728}$ ounces.

Exercises.

1. Find the pressure at the depth of 30 metres in a lake.—*Ans.* 3,000 grammes on a square centimetre.

2. Find the pressure on a square inch at a depth of 600 feet in a lake, the weight of a cubic foot being 1000 ozs.—*Ans.* 4166⅔ ozs.

3. In the experiment illustrated in Fig. 9. if the section of the cylinder be 36 square centimetres in area, and the weight of the disc 1080 grammes, to what depth must it be sunk before the thread may be dropped?—*Ans.* 30.

4. Find the pressure on a surface, 18 units by 15, at a depth of 20, the weight of a cubic unit of water being 10 lbs.—*Ans.* 54,000 lbs.

5. Two cubical vessels, the edges of which are as two to one, are filled with water, compare the pressure on the bases.—*Ans.* 8 to 1.

6. A cubic inch of mercury weighs 8 ozs., what will be the pressure on the bottom of a vessel, the area of which is a square foot, the depth below the surface being 5 inches?—*Ans.* 360 lbs.

7. A plate of metal is held at the end of a hollow glass cylinder 6 centimetres in diameter, while the cylinder is immersed vertically. It is found that the plate is just supported at a depth of 35 centimetres; find the weight of the plate in grammes.—*Ans.* 990 grammes.

8. If two vessels have equal bases and the same depth, but one widens upwards to the mouth, and the other diminishes upwards to the mouth, on which is the pressure on the base the greater?

9. What will be the pressure on a rectangle 11 by 60 inches, immersed so that a diagonal is vertical in a fluid a cubic inch of which weighs 1½ oz.?—*Ans.* 30,195 ozs.

10. What will be the pressure on a vertical triangle immersed in water with the base in the surface, the base being 50 centimetres, and height 30?—*Ans.* 7500 grammes.

11. The upper surface of a vessel filled with water is a square whose side is 8 metres, and a pipe communicating with the interior is filled with water to a height of 2 metres above the surface; find the weight which must be placed on the lid of the vessel to prevent the water from escaping.—*Ans.* 128,000 kilogs.

12. A house is supplied with water from a reservoir, 241 feet above the ground floor, by means of a pipe laid underground, what will be the pressure per square inch on a tap 25 feet above the

ground floor, supposing a cubic foot of water to weigh 1000 ozs.?—*Ans.* 1500 ozs.

13. A vessel is filled with water, how will the pressure on the base be affected if a piece of metal be dipped into the water?

14. If the vessel be not full, how will the pressure on the base be affected by dipping a piece of iron into the water?

IV.—THE PRESSURE ON A SOLID IMMERSED IN A LIQUID.

29. When bodies are placed in a fluid, it is a matter of common observation that they will sink or float, according as they are, bulk for bulk, heavier or lighter than the fluid. Even when a body sinks, it appears to be of less weight in water than in air.

For example, it is easy for a person to raise a mass of stone under water which he is unable to lift above the surface.

The pressures exerted by the fluid on the surface of the body have a resultant which acts in a direction opposite to that of the weight of the body. If the weight of the body exceed the fluid pressure, the body sinks; if the pressure be greater than the weight, the body ascends; when the weight and the pressure are equal, the body rests. These different conditions may be made to succeed each other at pleasure, with the toy represented in Fig. 11. A hollow globe of glass, containing air and water, floats

on the surface of the water in a glass cylinder, over the top of which is stretched an elastic cover. In the lower part of the globe there is a very small hole, by which water can enter or escape. If the hand be placed on the elastic top, the air in the cylinder is compressed, and its pressure on the surface of the water is increased. This augmented pressure is transmitted through the water to the air in the globe, which is condensed. The consequence is that a small quantity of water enters the globe, increases its weight, and causes it to sink. When the pressure is removed, the bulb rises again. The pressure may be so regulated that the bulb shall rest suspended in any part of the water.

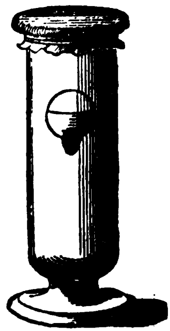

Fig. 11.

30. The law according to which a fluid exerts a pressure on a body immersed is as follows:—

When a solid is immersed in a liquid, the resultant pressure on it is the weight of its own bulk of liquid.

Let A be a solid immersed in water (Fig. 12). Imagine the space A filled with water, and then suppose the mass A to become solid; it will remain suspended like any other part of the fluid. Since its weight acts downwards, it is evident that there is an equal force acting upwards, counteracting the weight. This force is the resultant of all the fluid pressures, acting in all directions, upward,

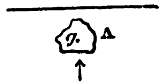

Fig. 12.

downward, and laterally. Some of these pressures neutralize others; but there must remain a balance of upward pressure, since the under surface is at a greater depth than the upper, and it is this resultant pressure which counteracts the weight of the imagined isolated mass. Therefore the upward pressure on A is equal to the weight of A. Since two forces in equilibrium must act in the same straight line, the resultant must pass through the centre of gravity, g, of the mass.

If we imagine the ice to have become stone without altering its bulk or its surface, the pressure acting upon it will still be the same, namely, the weight of the same bulk of water as would exactly fill the space occupied by the stone. Thus the weight of the body is diminished by the weight of the water displaced.

V.—SPECIFIC GRAVITY.

31. Equal bulks of different substances have different weights. If equal volumes be taken, lead is heavier than iron, iron is heavier than water.

It is a problem of great practical importance to determine the relation between the weights of different substances. For this purpose all substances are compared with water, and the relative weight of any substance is expressed by the number of times the weight of the given body contains the weight of

an equal bulk of water. This number of times is called the specific gravity of the body.

32. *The measure of the specific gravity of a body is the ratio of the weight of any volume of the body to the weight of the same volume of a certain standard substance.*

Water by its universal distribution and the ease with which any required volume is obtained and purified, is best adapted for the standard substance.

Hence if s be the specific gravity of a body, the weight of the body over the weight of an equal bulk of water$=s$.

Let V be the volume of the body, and let w be the weight of a unit of water, then the weight of the same bulk of water$=w\mathrm{V}$, thus :—

$$\frac{\mathrm{W}}{w\mathrm{V}} = s \text{ or } \mathrm{W} = w\,\mathrm{V}s$$

If the unit of volume be a cubic centimetre of distilled water at 4° w=one gramme; if the unit of volume be a cubic foot w=1000 ozs. nearly.

Thus, if we know the specific gravity of a body we can find the weight of any volume.

Example 1.—What is the weight of 48 cubic inches of copper the specific gravity of which is 8·82.

Weight of 48 cubic inches of water$=\dfrac{48\times1000}{1728}$

Hence, weight of 48 cubic inches of substance, whose specific gravity is 8·82

$$=\frac{48\times1000\times8{\cdot}82}{1728}=245 \text{ ozs.}$$

Example 2.—What is the weight of 250 cubic centimetres of mercury, the relative weight being 13·596?

1 cubic centimetre of water weighs 1 gramme,
1 cubic centimetre of a substance of specific gravity 13·596 weighs . . . 13·596 grammes,
250 cubic centimetres weigh $250 \times 13{\cdot}596$ grammes,
Or 3399 grammes.

Example 3.—What volume of lead of specific gravity 11·4 will weigh 190 grammes?

11·4 grammes of lead have the same bulk as 1 gramme of water.

Hence 11·4 grammes of lead measure 1 cu. cent.

1 gramme " " $\frac{1}{11{\cdot}4}$ "

190 grammes " " $\frac{190}{11{\cdot}4}$

Or $16\frac{2}{3}$ cubic centimetres.

Example 4.—A piece of marble of specific gravity 2·7 weighs half a ton. What is its size?

Half a ton=17,920 ozs.

Volume of 1000 ozs. of water = 1 cu. ft.
" 1 oz. " = ·001 "
" half a ton " = $17920 \times {\cdot}001$ cu. ft.
= 17·92 cu. ft.

Volume of the same weight of a substance whose sp. gr. is 2·7 $\Big\} = \frac{17{\cdot}92}{2{\cdot}7} = 6{\cdot}06$ cu. ft.

Example 5.—Find the specific gravity of a mixture of 1 cubic centimetre of a fluid of specific

gravity 1·3, and two cubic centimetres of specific gravity 2·2.

Weight of 1st, 1·3 grammes.

Weight of 2nd, twice 2·2 or 4·4

Weight of mixture, 5·7 grammes.

Volume of mixture=3 cubic centimetres, and weight of equal bulk of water=3 grammes.

Hence specific gravity of mixture$=\frac{5\cdot7}{3}=1\cdot9$.

Example 6.—To find the specific gravity of a mixture of given volumes of any number of given fluids.

Let V_1 V_2 V_3 etc. be the volume of the fluids, let s_1 s_2 s_3 be their specific gravities. Then the weight of the mixture is $w\,(s_1V_1 + s_2V_2 + s_3V_3 + \text{etc.})$

$\therefore$ Weight of same volume of water is $w\,(V_1+V_2+V_3+\text{etc.})$, and consequently the specific gravity of the mixture is

$$\frac{s_1V_1 + s_2V_2 + s_3V_3 + \text{etc.}}{V_1 + V_2 + V_3 + \text{etc.}}$$

Example. 7.—To find the specific gravity of a mixture where the weight and specific gravity of the compounds are known.

Let w_1 w_2 w_3 etc., be the weights, let s_1 s_2 s_3 be the specific gravities.

The volumes are $\frac{w_1}{s_1}$, $\frac{w_2}{s_2}$ etc. Hence, substituting in the last exercise, we have the specific gravity of the mixture

$$=\frac{w_1 + w_2 + w_3 + \text{etc.}}{\frac{w_1}{s_1} + \frac{w_2}{s_2} + \frac{w_3}{s_3} + \text{etc.}}$$

Exercises.

1. A body measuring 18 cubic centimetres floats in water with its whole bulk immersed; find its weight.

2. Define the specific gravity of a body. If 150 cubic centimetres of a body weigh 120 grammes; find its specific gravity.—*Ans.* ·8.

3. If 200 cubic centimetres of brass have a specific gravity of 7·5, find the weight.—*Ans.* 1500 grammes.

4. Having given that 10 cubic feet of stone weighing 24,950 ozs., have a specific gravity of 2·5, find the weight of a cubic foot of water.—*Ans.* 998 ozs.

5. What is the weight of 12 cubic inches of mercury, the specific gravity of which is 13·5, if a cubic foot of water weighs 1000 ozs.—*Ans.* 93·75 ozs.

6. What volume of zinc, of specific gravity 6·85, will weigh 109·6 grammes?—*Ans.* 16 cubic centimetres.

7. What is the size of a block of stone, the weight of which is 1 cwt., and specific gravity 2·4?—*Ans.* ·746 cubic feet.

8. Three liquids, the specific gravities of which are respectively 1·2, ·96, and 1·456, are mixed in the proportions of $1\frac{1}{2}$ gals. of the first to $1\frac{1}{3}$ gals. of the second, and $1\frac{1}{4}$ gals. of the third; find the specific gravity of the mixture.—*Ans.* 1·2.

9. What will be the weight in water of 100 cubic centimetres of iron, the true weight being 784 grammes.—*Ans.* 684 grammes.

10. A cubic centimetre of alcohol weighs ·79 grammes, what will be the true weight of a body which floats in alcohol with 1000 cubic centimetres of its volume immersed?—*Ans.* 790 grammes.

11. What is the weight of a cubic centimetre of a body which floats in water with one-fifth of its volume above the surface?—*Ans.* ·8 grammes.

12. Find the weight of a cubic foot of mercury, the specific gravity of which is 13·568, supposing a cubic foot of water to weigh 1000 ozs.—*Ans.* 848 lbs.

13. If a cubic unit of the standard substance weigh ·35, what is the weight of 1000 cubic units of substance whose specific gravity is 3?—*Ans.* 1050.

14. The specific gravity of two liquids are respectively 1·8 and

·85. A measure of the former is added to three measures of the latter; find the specific gravity of the mixture.—*Ans.* ·9625.

15. Ten cubic centimetres of a substance weighs $\frac{1}{16}$ kilogs.; find the specific gravity of the substance.—*Ans.* 6·25.

16. Equal volumes of three fluids are mixed, the specific gravity of the first is 1·55, that of the second is 1·75, and that of the mixture is 1·6; find the specific gravity of the third.—*Ans.* 1·5.

VI.—TO FIND THE SPECIFIC GRAVITY.

33.—To find the relative weight or specific gravity of a body, we require to know its weight and the weight of an equal bulk of water. We will now point out methods of obtaining the weight of water equal in bulk to a given body. We have seen that when a solid is immersed in a fluid, it displaces its own volume of the fluid. Suppose that we immerse the body gently in a vessel exactly filled with water, and weigh the water which overflows, we shall then have the required weight of water.

34. A more convenient method is afforded by the principle that the solid, when weighed in a fluid, appears to be lighter by the weight of the fluid displaced. This law is termed the principle of Archimedes. It is said that Hiero, King of Syracuse, applied to Archimedes for a test to prove whether a crown which had been made by his orders

was all gold, or whether the goldsmith had dishonestly substituted a baser metal for a portion of the gold. While the philosopher was thinking of the subject, he chanced to enter a bath filled with water, and noticed that, as he entered, the liquid flowed over.

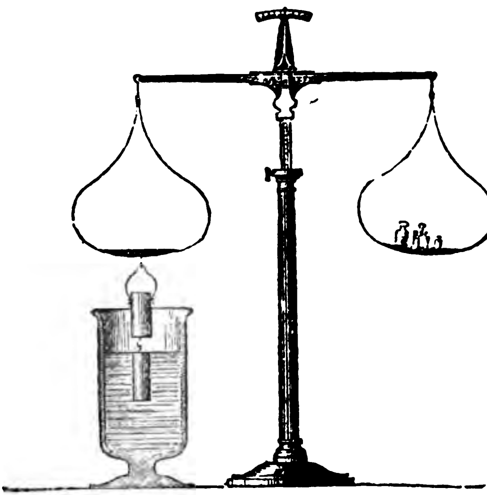

Fig. 13.

This observation suggested a solution of his problem. He took the crown, and a quantity of pure gold of the same weight, and immersed them successively in the same vessel, filled to the brim with water. As the crown displaced more water than the equal bulk of gold, he concluded that it was partly composed of a

lighter metal, and the king's suspicions were confirmed.

35. The principle of Archimedes may be tested experimentally thus:—Two cylinders, the first *a* hollow, and the second *b* solid, and of such a size as exactly to fill the first, are suspended from the scale of a balance and weighed. The cylinder *b* is then suspended from *a* and allowed to dip into a vessel of water. The opposite scale then goes down; but it is found that equilibrium is restored when the cylinder *a* is filled with water.

A Solid heavier than Water.

36. Hence, to find the specific gravity of a solid heavier than water, and not soluble in it, weigh the body in air, then weigh it in water; the specific gravity

$$=\frac{\text{the true weight}}{\text{loss of weight in water.}}$$

Example.—A piece of metal weighs 18 kilogs. in air, and 15 in water; find its specific gravity.

The loss of weight in water$=18-15=3$ kilogs.

Hence the specific gravity $=\frac{18}{3}=6$.

37. An instrument termed Nicholson's hydrometer may be employed to compare the weight of a body with that of an equal bulk of fluid.

It consists of a hollow cylinder *c* (Fig 14), supporting a light cup *b* on a thin stem above it, and a heavy cup *h* below it. The whole floats in the fluid.

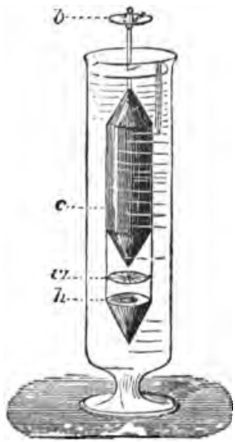

Fig. 14.

A certain weight (suppose 10 grammes) is placed in the upper cup, and a mark is made on the stem, to indicate the surface of the liquid. The body is then placed in the cup; the hydrometer will descend, in consequence of the additional weight. Such weights are then taken off, as will cause the instrument to rise until the mark made is again at the surface. The weights taken off will be the weight of the body. For example, let us suppose 7 grammes removed, then this is the weight of the body. Now remove the body to the lower cup; the instrument ascends. Place such a weight on it as will bring it to the same level. The weight added is the loss of weight in water. Suppose 3 grammes added, then the specific gravity

$$= \frac{7}{3} = 2\tfrac{1}{3}.$$

To Find the Specific Gravity of a Solid Lighter than Water.

38. 1*st Method.*—If a wire gauze *w* be stretched over the lower cup in Nicholson's hydrometer, so as

to prevent a light body rising, the method last described will suit this case also.

2nd Method.—The specific gravity of a solid lighter than water may also be found by attaching a heavier body to it.

Example.—A block of wood, weighing in air 8 lbs., is tied to a piece of iron weighing 6 lbs.; the whole weighs in water 4 lbs., and the iron only weighs 5 lbs. Find the specific gravity of the wood.

The weight of water equal in bulk to the iron . $=6-5=1$

The weight of water equal in bulk to both iron and wood $=(8+6)-4=10$

$\therefore$ Weight of water equal in bulk to the wood . $=10-1=9$.

Hence the specific gravity of the wood $=\frac{8}{9}$. And in every case the specific gravity of the lighter body

$$=\frac{\text{weight of light body}}{\text{loss of weight of both} - \text{loss of weight of heavy body.}}$$

To find the Specific Gravity of a Liquid.

39. *1st Method.*—Weigh a flask filled with the liquid, then weigh the flask filled with water; deduct from each result the weight of the flask, then the specific gravity of the liquid is its weight, divided by the weight of the water.

Example.—A flask, when empty, weighs 2 ozs., when filled with alcohol it weighs 6 ozs., and when

Example.—A flask weighing 10 grms. when empty, weighs 10·6465 grms. when filled with air, and 510 grms. when filled with water. Find the relative weight of air.

The air which fills the flask weighs ·6465 grms.
And the water 500 grms.
The relative weight is therefore

$$\cdot 6465 \div 500 = \cdot 001293.$$

To find the Specific Gravity of a Solid Body Soluble in Water.

41. Weigh the body in air, and then in some liquid in which it is insoluble. The loss of weight is the weight of a quantity of this liquid having the same bulk as the body. The weight of the same bulk of water is found by dividing the result by the specific gravity of the liquid.

Example.—A piece of sodium weighs 8 grms. in air, and 1·07 grms. in naphtha, the specific gravity of which is known to be ·84. Find the specific gravity of the sodium.

The weight of the naphtha having the same volume as the sodium $= 8 - 1\cdot07$, or 6·93 grms.
The weight of the same bulk of water=

$$6\cdot93 \div \cdot84, \text{ or } 8\cdot25.$$

Hence the specific gravity $= \dfrac{8}{8\cdot25} = \cdot97.$

Instead of definite weights, let us take general

symbols. Let w_a=the weight of the sodium in air, w_n=the weight in naphtha, and let S=the specific gravity of the naphtha; then

$$\text{the specific gravity}=\frac{w_a}{(w_a-w_n)\div S}=\frac{w_a \cdot S}{w_a-w_n}.$$

Hence we may find the specific gravity referred to water by finding first the specific gravity referred to some other liquid, and then multiplying by the specific gravity of this liquid.

42. We may also find the specific gravity of a liquid by comparing the amount displaced by a body made to float in the liquid and in water. Fig. 15 represents such a body, termed a common hydrometer. It is usually made of glass, and consists of a tube A, a large bulb B, and a small one C, loaded so as to keep the stem vertical.

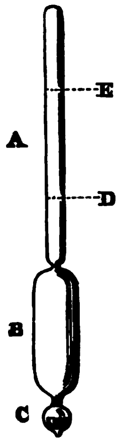

Fig. 15.

Let w be the weight of the instrument, and a the area of a section of the stem. Suppose the instrument to sink to D in water, and to E in the other liquid, then if v be the volume below D, and S the specific gravity of the fluid, $v \div S$ is the volume below E. But since the volume of the part DE is $a \,.\, \text{ED}$:

$$\therefore \frac{v}{S}=v+a \,.\, \text{ED}.$$

43. In the preceding examples we have not taken account of the weight of the air displaced by the solid weighed in it. The true weight of the solid is the weight in air, together with the weight of the air displaced. We may find approximately the weight of the air displaced from the weight of the water displaced. At the ordinary pressure of the atmosphere 1 litre of water weighs 1000 grms., and 1 litre of air 1·293 grms.; hence the weight of air displaced is ·001293 of the water displaced.

Example.—A body weighs 8 kilogs. in air, and 5 in water; find its specific gravity, taking into account the weight of the air. The weight of water displaced is very nearly 8 — 5, or 3 kilogs. This is not quite true, because 8 is not quite the true weight of the body. Hence the weight of the air is very nearly ·001293 × 3000 grms., or 3·879 grms. Hence the specific gravity is

$$\frac{8003 \cdot 879}{3003 \cdot 879} = 2 \cdot 664545.$$

Density.

44. The density of a body is a term for the ratio of the quantity of matter in the body to the quantity of matter in an equal bulk of a standard substance. Thus, if water be the standard, the density

$$= \left(\frac{\text{mass of the body}}{\text{mass of an equal bulk of water.}}\right)$$

The density, therefore, refers to the mass as the specific gravity refers to the weight of a body.

If we suppose the force of gravity constant, as is the case at any one particular spot on the earth's surface, the weight will be proportional to the mass, and the density and specific gravity of a body will be the same. The variation of the force of gravity does not affect the density, but it does affect the specific gravity.

From the above formula we see that
if the volume be kept the same
the density varies as the mass,
and if the mass remain the same
the density varies inversely as the volume.
Hence, when both volume and mass vary,

$$\text{the density varies as } \frac{\text{mass}}{\text{volume}}$$

Therefore $d = \dfrac{C\,M}{V}$ where c is a constant.

It has been shown in "Mechanics" (p. 175) that writing M for mass, W for weight, and g for acceleration due to gravity,

$$\begin{array}{llll} & W \text{ varies as } M & \text{when } g & \text{is constant,} \\ & W \quad \text{,,} & g & M \quad \text{,,} \\ \therefore & W \quad \text{,,} & Mg & \text{both vary} \\ & & \therefore W = CMg, & \end{array}$$

but from the above formula

$$M = dV \times \text{ a constant}$$
$$\therefore W = cdgV$$

The units may be so chosen, therefore, that

$$W = gdV.$$

Problems on the Density of Liquids.

45. If two liquids that do not mix meet in a bent tube, their heights above their common surface will be inversely proportional to their densities. Let A be the upper surface of the denser fluid of density d, B that of the lighter fluid of density d_1, and let C be their common surface (Fig. 16). Let h = the height of A above the horizontal plane C D and h_1 = height of B above C D.

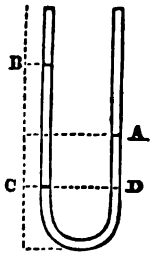

Fig. 16.

The pressure at C $= gd_1h_1$ the pressure at D $= gdh$, and these are equal, since they are in the same horizontal plane.

$$\therefore gdh = gd_1h_1$$

$$\text{or } h : h_1 = \frac{1}{d} : \frac{1}{d_1}.$$

EXAMPLES.

1. A body suspended by a thread from one scale of a balance weighs 36·84 grammes; a vessel of water is placed so that the body is suspended in the water, and then 6·21 grammes must be placed in the scale above the body to restore equilibrium; find the specific gravity of the body.—*Ans.* 5·9.

2. Find the specific gravity of a piece of lead which weighs 47·48 grammes in air and 43·33 grammes in water.

3. The specific gravity of aluminium is 2·56, and a fragment when weighed in water loses 3·78 grammes. What is its weight? —*Ans.* 9·6768 grammes.

4. A body weighs 5 grammes; when the body and 100 grammes of lead of specific gravity 11·44 are weighed together in water, the loss of weight is 9·5 grammes. What is the specific gravity of the body?—*Ans.* 6·59.

5. A cannon made of metal of specific gravity 8·55 weighs 246 kilogs. in air, and 210 kilogs. in water. Is there a flaw in the cannon? What is its volume?—*Ans.* There is a flaw; the volume = 36,000 cubic centimetres.

6. A body weighs 25 grammes in air, 20 grammes in water, and 15 grammes in another liquid; find the specific gravity of the body and of the second liquid.—*Ans.* 5 and 2.

7. A balloon weighs 263·525 grammes when empty, and 379·895 grammes when filled with air at the ordinary pressure. What is its capacity, the weight of a litre of the air being 1·293 grammes?—*Ans.* 90 litres.

8. The same balloon, when filled with gas under the same conditions of temperature and pressure, weighs 293·687. What is the specific gravity of the gas referred to air as the standard?—*Ans.* ·26 nearly.

9. A body floats with one-third of its bulk under water; what is its specific gravity?

Let W = weight of body; then ⅓ of its bulk of water = W, and the weight of its own bulk of water is therefore 3 W. The specific gravity is therefore W ÷ 3 W = ⅓.

10. A piece of gold weighs 9·7 grammes, a flask full of water weighs 95 grammes; the gold is put into the flask, and displaces a small quantity of the water, which flows over. The weight of the whole is 104·2 grammes; find the specific gravity of the gold.—*Ans.* 19·4.

11. Nicholson's hydrometer is used as follows:—51·72 grammes is placed on the upper cup, a fragment of metal is placed on the cup, and it is found necessary to take off 14·85 grammes to make the instrument float at the same level; the metal is then placed in the lower cup, and 2·03 grammes added above to restore the former level. Find the specific gravity of the metal.—*Ans.* 7·31.

12. A globe of glass weighs in pure water 6·9 grammes, in sea water 6·2 grammes, and in air 13 grammes. Find the specific gravity of the sea water.—*Ans.* 1·115.

13. A cylindrical piece of cork of specific gravity ·25 and length of 10 inches floats upright in water; how much of it will be immersed?—*Ans.* 2·5 inches.

14. A piece of wood weighs 7 lbs. and a piece of iron weighs 7·8 lbs. in air and 6·7 lbs. in water; the wood and iron together weigh 5·3 lbs. in water. What is the specific gravity of the wood?—*Ans.* ⅞.

15. A body weighs 10 ozs. more in air than in water; if 1 cubic foot of water weighs 1000 ozs., what is the volume of the body in cubic feet?—*Ans.* ·01.

16. A cylinder floats in a fluid of specific gravity 2·5 with two-thirds of its bulk above the surface. What is the specific gravity of the cylinder?—*Ans.* ·83.

(The specific gravity referred to the liquid $= \frac{1}{3}$ $\therefore$ referred to water $= \frac{1}{3} \times 2{\cdot}5$.)

17. If the cylinder be removed to another fluid, and then floats with one-third of its bulk out, find the specific gravity of the fluid. —*Ans.* 1·25.

18. A vessel containing water is placed in one scale of a balance, and counterpoised by a weight placed in the other scale. If 10 cubic centimetres of metal be suspended in the water so that none of the water is spilt, what additional weight must be placed in the other scale to restore equilibrium?—*Ans.* 10 grammes.

19. To one scale of a balance a weight of 20 lbs. and specific gravity 8·5 is suspended, and allowed to rest in water, what weights must be put in the other scale to restore equilibrium?

Solution. 8·5 lbs. of metal lose 1 lb. in water.

„ „ weigh 7·5 lbs. in water.

1 lb. „ weighs $\dfrac{7{\cdot}5}{8{\cdot}5}$ „

20 lbs. „ „ $\dfrac{20 \times 7{\cdot}5}{8{\cdot}5} = 17\frac{11}{17}$

20. A rod of wood floats with one-fourth of its bulk out of water, and a stone equal in bulk to one-tenth of the rod, when placed on the rod, sinks it until the top of the rod is level with the surface. Find the specific gravity of the stone.—*Ans.* 2·5.

21. A body whose specific gravity is unity is placed in water, how much weight does it lose?

22. A vessel and water weigh 10 lbs., what will they weigh when a body which floats with 5·76 cubic inches immersed is placed in the water.—*Ans.* $10\frac{5}{24}$ lbs.

23. Two bodies, *a* and *b*, of different bulks weigh the same in water, which will weigh the most in mercury?—*Ans.* The smaller.

24. Which will weigh the most in air?—*Ans.* The larger.

VII.—THE AIR AND GASES.

46. Many of the properties of liquids are also possessed by elastic fluids. For example, the air has weight. To prove this, let us take a flask fitted with an air-tight stop-cock, and by means of an instrument to be hereafter described, let us pump all the air out of it, and turn off the stop-cock. Let the flask be placed in the scale of a balance, and such weights in the other as will keep the beam horizontal. When the stop-cock is turned on, the air will be heard to rush in and the flask will descend. If, instead of the flask, a glass globe capable of containing a cubic foot be emptied and weighed, when the air is admitted it will be an ounce and a half heavier than before. Again, pressure is transmitted by liquids and gases according to the same laws; for example, the pressure at a point in a gas is the same in all directions.

47. A very simple experiment will prove the existence of atmospheric pressure. Fill a tumbler with water, and place a sheet of paper over the top, hold the palm of the hand against the paper, and invert the tumbler. The hand may now be withdrawn without the water falling (Fig. 17). The atmospheric pressure supports the water, the paper being used only to prevent the air from replacing them in the glass.

Fig. 17.

48. If a glass cylinder be tightly closed at one

end by an air-tight membrane, and the air be withdrawn from the interior by means of an air-pump, the pressure of the atmosphere, not being supported by the elastic force of the air inside the cylinder will burst the membrane.

49. Nothing in the history of science is more remarkable than that up to the time of Galileo (1636) the fact that the atmosphere exerted pressure was unknown. Effects produced by this pressure were observed on all sides, but they were set down to other causes. When the water rose in a pipe from which the air had been withdrawn, as is the case with the common pump, in apparent violation of the laws of gravity and of the law that liquids maintain their levels, philosophers, unable to account for the circumstance, satisfied themselves by considering it due to a freak of nature. They set it down as an axiom that "nature abhors a vacuum," that nature would not allow any part of the universe to be void of matter. Thus, they argued, when air has been withdrawn from the pipe of a pump, nature's abhorrence of a vacuum compels the water to rise and fill the vacant space.

In 1636, however, some mechanics made a pump to raise water from a well, the surface of the water being 50 feet deep. They found that they could not make the water rise more than 32 feet. They applied to the celebrated philosopher, Galileo, to solve the mystery. He could not explain it, but he said one thing was certain—Nature's abhorrence of a vacuum did not extend over 32 feet.

The subject was then investigated by Torricelli

a pupil of Galileo. He argued that whatever the cause might be, since it was sufficient to support 32 feet of water, if a heavier liquid were used, a column of less altitude should be supported. For example,

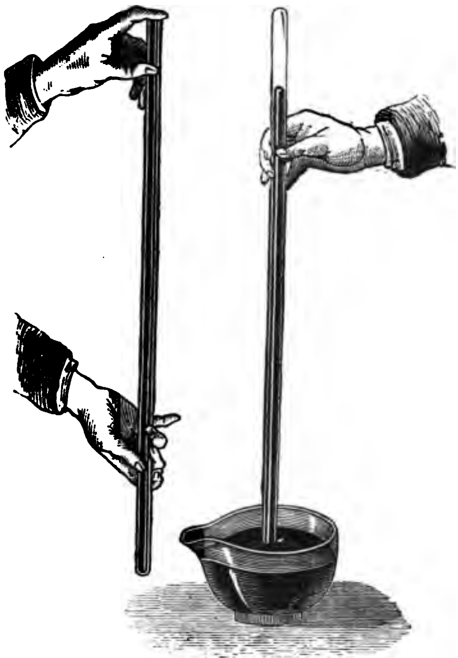

Fig. 18.

since 32 feet of water is sustained, of mercury, which is $13\frac{1}{2}$ times as heavy as water, 32 feet divided by $13\frac{1}{2}$, or 28 inches, ought to be supported.

He conducted his experiment as follows:—He took a vessel of mercury and a glass tube over 30

inches long, open at one end and closed at the other. Having filled the tube with mercury (Fig. 18), he closed the end with his finger, inverted the tube, plunged the covered end below the surface of the mercury in the vessel, and removed his hand.

The mercury in the tube subsided until it stood at the height of 28 inches.

Supposing the surface of the mercury in the vessel to be prolonged under the tube, he saw that the fluid outside the tube was only subjected to the pressure of the atmosphere, while that under the tube sustained the column of mercury above it. The secret was now revealed to him, for he concluded at once that the pressure of the atmosphere on an area equal to the aperture of the tube was equal to the weight of the mercury in the tube.

This conclusion was confirmed by carrying the apparatus up a mountain. Pascal, a French divine, argued that when a portion of the air was left below, the pressure of that which remained must be diminished; consequently, if the column of mercury were really supported by this pressure, the height of the column must vary with different distances above the earth's surface. The experiment was tried, and the mercury in the tube was found gradually to sink during the ascent.

Magdeburg Hemispheres.

50. A few years later (in 1654) a Professor in Magdeburg constructed an apparatus which not only

exhibited the force of the atmospheric pressure, but showed that this pressure acted equally in all directions. The apparatus consists of two hollow metallic hemispheres, the edges of which are made to fit exactly (Fig. 19). To one of the hemispheres there is a tube and stop-cock, so that when the two are placed together the air may be pumped out. When filled with air the hemispheres may be separated without difficulty, but, when the air is exhausted, in whatever position they are held, a powerful effort is required to separate them.

Fig. 19.

The Barometer.

51. The apparatus used in Torricelli's experiment constitutes a barometer. Whatever the size of the tube, the pressure at the base of the column on a level with the surface of the liquid outside must be equal to the pressure of the air on the same area anywhere in that surface.

Thus, if the area of a section of the tube be one square inch, the pressure on a square inch is the weight of the mercury in the tube; if the section be a tenth of a square inch, the weight of the column of mercury above the surface is the pressure of the air on the tenth of a square inch, and ten times this weight is, of course, the pressure on a square inch.

It is easy, therefore, to find by means of this experiment the pressure of the air. Suppose, for example, a tube is used having a section exactly one

square centimetre in area, and suppose the column of mercury to stand at the usual height of 76 centimetres; there will then be 76 cubic centimetres of mercury in the tube. Now, the specific gravity of mercury is 13·596, hence 76 cubic centimetres of mercury weigh 13·596 grammes $\times$ 76 = 1033 grammes. A pressure of 1033 grammes on a square centimetre is termed a pressure of one atmosphere.

If we take inches and ounces as the units, supposing the height of the barometer to be 30 inches, and the weight of a cubic inch of water $\frac{1000}{1728}$ ozs., the atmospheric pressure = $30 \times 13{\cdot}596 \times 1000 \div 1728$ ozs. = 14·7 lbs.

Marriotte's Law.

52. On account of the elasticity of a gas, the pressure exerted by it changes with its temperature and with its volume.

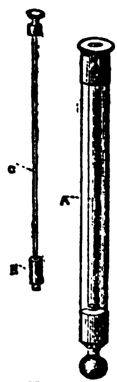

Fig. 20.

For example, take a cylindrical vessel fitted with an air-tight piston (Fig. 20). When held vertical, the piston will be supported by the resistance of the air within the cylinder. An effort is required to force in the piston, and the more the air is compressed the greater the pressure required to keep the piston from returning.

Instead of a force applied by the hand, let us place a weight on the piston. Suppose the pressure of the air as indicated by the barometer to be 15 lbs. on the square

inch. Suppose, for convenience of reckoning, we take a cylinder A B, of 1 square inch in area, and 12 inches long. Before the piston is inserted the air in the cylinder sustains a pressure of 15 lbs. Put in the piston, and place on it such weights as will cause it to descend to A, the middle of the cylinder. The air will have been compressed into half its bulk, and it will be found that the weight amounts to 15 lbs. Then double the pressure reduces the volume one half. If we make the piston and its load twice 15 lbs., so that the whole pressure is thrice 15 lbs., the piston will descend to E, one-third of the height. When the pressure is four times 15 lbs., the bulk is one-fourth of that at a pressure of 15 lbs. A pressure of 12 atmospheres will reduce the air to one-twelfth of its original volume.

This relation of the pressure to the volume of a gas is termed Marriotte's Law.

"If the temperature be kept the same, the pressure of a quantity of air or gas is inversely proportional to the space it occupies."

This law was proved by Boyle and Marriotte as follows:—A bent tube A B with open ends (Fig. 21), and one arm A longer than the other, is fixed to a graduated stand, and a little mercury is poured into it. The surface C C′ of the mercury will be in the same horizontal plane, and will be subjected in both tubes to the pressure of the atmosphere. The end B is now closed, and more mercury poured into A. The mercury rises to D in the arm B, compressing

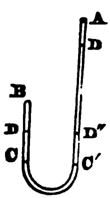

Fig. 21.

the air, and to D′ in the arm A. The pressure of the mercury in the two arms below the same level D D″ balance each other, and the pressure at D″ is therefore that of the atmosphere, added to the weight of the column D′ D″.

Suppose P_1 be written for the original pressure on the air in B, and P_2 for the pressure after compression; suppose that the pressure of the air would support the mercury at a height h, and let the weight of mercury in one inch of the tube be w,

then the pressure of the atmosphere $= hw$,

and that of the atmosphere and the column $D'D'' = (h + D'D'')w$,

hence $\dfrac{P_1}{P_2} = \dfrac{h}{h + D'D''}$

Now, when the scale is applied to the spaces BC, BD, it is always found that $\dfrac{BD}{BC} = \dfrac{h}{h + D'D'}$

hence $\dfrac{BD}{BC} = \dfrac{P_1}{P_2}$

The relation between pressure and volume may be also expressed thus:—If the temperature remains the same, the volume multiplied by the pressure remains the same. Thus, if V be the volume when the pressure is P, and V^1 the volume when the pressure is P^1, $VP = V^1P^1$.

Relation of Pressure and Temperature.

53. The pressure of a gas is affected by change of temperature.

If the heat syringe (Fig. 20), with a certain load

on the piston, be placed in warm water, the piston will be forced up unless additional weight be added.

If the pressure remain the same, an increase of temperature of 1° centigrade produces in a given mass of gas an expansion of ·003663 of the volume it would have at 0° centigrade.*

The Weather-Glass.

54. The pressure of the atmosphere not only varies with the height above the sea, but it changes at the same place within certain limits; consequently, the height of the mercury in the barometer-tube is subjected to frequent variations. These variations at the level of the sea range from 785 to 731 millimetres, or from 31 to 28 inches.

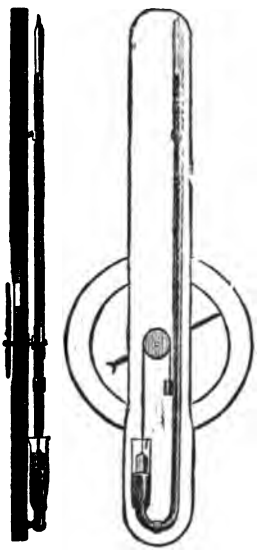

Fig. 22.

A change in the height of the barometer frequently coincides with a change of the weather. In our climate it is a matter of observation, that in rainy and stormy weather the barometer is usually below the average height, and in fine and dry weather the

* For explanation and examples of this law, see Orme's "Heat," page 64.

height of the barometer is usually above the average. Hence the barometer is used as a weather-glass.

The form of instrument used for this purpose is represented in Fig. 22. It consists of a siphon barometer, in the shorter leg of which is a float, which rises and falls with the mercury. The float is connected, by some mechanical contrivance, with a wheel, bearing a needle. When the pressure of the air varies, the sinking or rising of the float causes the needle to move over a graduated scale in front of the case enclosing the barometer.

The Common Pump.

55. The pressure of the atmosphere is taken advantage of for the purpose of raising water above its level.

The common pump is a machine by means of which this is effected. A B is a cylinder, having its lower end closed with a valve B (Fig. 23) opening upwards, and connected by means of a pipe with the water which is to be raised. A piston containing either one or two valves opening upwards, is worked in the cylinder by means of a vertical rod and a handle.

Suppose the piston to be in its lowest position, then, when it is raised, the valve B opens, and a partial vacuum is produced in the cylinder and pipe; and the pressure of the atmosphere without being greater than the pressure within the pipe, the water rises, and it continues to rise until the pressure within and without become equal.

When the piston descends the valve in the piston opens, and the air within the cylinder escapes. When the upward-stroke of the piston is complete, it is again depressed; the water passes through the valve in the piston, and on the next stroke it is discharged at the spout.

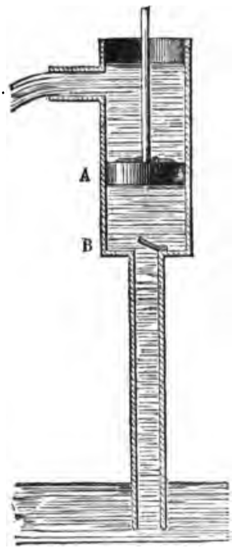

Fig. 23.

If the valves and pipes be perfectly air-tight, the greatest height to which the water could be raised by a common pump, reckoning from the surface of the water in the well to the bottom valve, is 33·75 feet, when the mercurial barometer is 30 inches, for

$$30 \text{ inches} \times 13{\cdot}5 = 33{\cdot}75 \text{ feet},$$

13·5 being the specific gravity of mercury.

The Lifting Pump.

56. Sometimes pumps are used to lift water to a great height. In this case the spout is connected with a vertical tube, and the water, instead of escaping by the spout, rises in the tube, and is prevented from returning by a valve.

The Archimedian Screw.

57. One of the earliest machines used for lifting water on record is the screw of Archimedes (Fig. 24). It consists of a cylinder inclined to the vertical, and exactly fitted by a screw having the same axis. If a small solid body were placed at the bottom, and the screw turned round, each point of the screw would

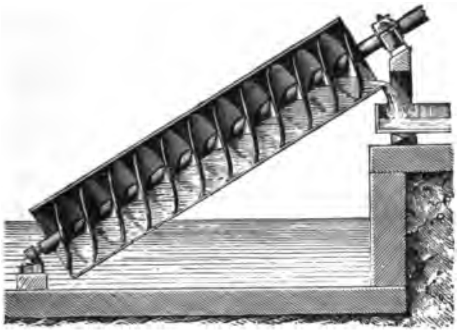

Fig. 24.

pass beneath the body at the lower edge of the cylinder, and the body would thus be raised gradually to the top. In the same manner, if water has access to the bottom, on turning the instrument it will be gradually raised until it flows out at the top.

The Forcing Pump.

58. The forcing pump differs from the common pump in the following respects: first, there is no spout or outlet to the upper part of the cylinder; secondly, the piston A is solid; thirdly, in addition to the pipe closed by the valve C, and communicating with the cistern, there is another pipe passing from

the lower part of the cylinder, with a valve c' opening upwards, through which the water is to be raised. ;

Suppose the piston to be in its lowest position when it is raised the valve c opens and c' closes, the column of water within the cylinder and pipe is relieved of the pressure above A, and the pressure of the atmosphere without being thus greater than that within the pipe, the water within rises, and continues to rise until the pressures within and without are equal. Let the piston be now made to descend, and the process repeated until the water has risen above the top of the pipe; then when A descends, the water in the cylinder, not being able to return on account of the valve c, is forced up the pipe, in which it is retained by the valve c'.

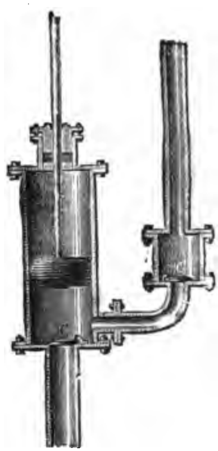

Fig. 25.

The Fire-Engine.

59. When a continuous stream is required, the valve leads into a strong air-vessel, out of which the pipe passes. After the down-stroke of the piston the air in the chamber is condensed, and its pressure on the surface continues to lift the water during the up-stroke. The fire-engine consists of two force-pumps connected with an air chamber (Fig. 26). The

pumps are worked by a lever, and are arranged so that when one piston descends and closes the valve below it, the other ascends and opens the valve communicating with the water in the cistern.

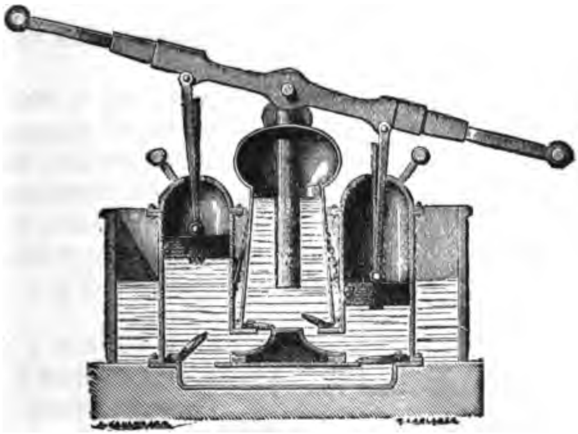

Fig. 26.

The Siphon.

60. The siphon is a bent tube A B C (Fig. 27) open at both ends. Let the tube be filled with fluid, and the shorter leg inserted into a vessel of fluid which it is required to empty, and the extremity C of the other leg closed by the finger. Let the level of the surface of the fluid meet the two legs of the siphon in G and D respectively. Provided the height of B above G D is not greater than that of a column of water, the weight of which is equal to the atmospheric pressure, the water in D B will be supported by the atmospheric pressure. Let P be the atmospheric

pressure, the pressure at D=P, and is transmitted along the tube.

The pressure at B=P—the pressure due to a column of water of height D E.

The pressure at G=P.

The pressure at C=P+the pressure due to a column of water of height g C.

Consequently the pressure on the end C, which we have supposed closed, exceeds the atmospheric pressure by the pressure due to a column of water of height of g C, or C F—D E.

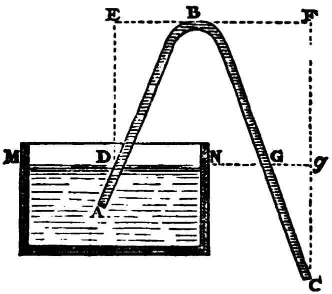

Fig. 27.

If the end C be opened the fluid will descend. Since at every point of the tube there is a pressure in the direction A B C, as soon as the liquid at any point moves forward, the particles behind being relieved of the pressure in front immediately follow, and thus a continuous stream is produced, which will only cease when the surface of the fluid has descended to A, the extremity of the shorter leg of the siphon.

If the leg B C terminated at G, it is evident that there would be equilibrium.

If the height D E were greater than that of the water-barometer, the liquid would descend in each tube to that height, leaving a vacuum at B.

The Air-Pump.

61. The air-pump is a machine constructed for removing the air from a closed vessel, called a receiver.

From the centre of a metal plate to which the receiver R is fitted (Fig. 28), so that no air can pass

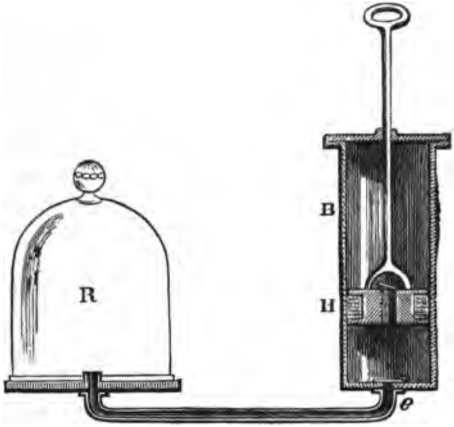

Fig. 28.

between them, runs a pipe communicating with the barrel of the pump B by means of a valve *e*, opening upwards. In the barrel is an air-tight piston H also furnished with a valve opening upwards.

Suppose H to be drawn up, then a partial vacuum will be formed below the piston, and fresh air rushing in from R will open the valve at *e*.

Now let H be made to descend, then the pressure of the air will close the valve *e* and open the valve in H, and the air in the barrel will be forced out. In

the meantime the air in the receiver expands and refills the barrel.

Now H again descending and ascending expels another portion, and so on till the air in R becomes so rare that its elastic force is insufficient to open the valves. It is obvious that, although the air gets rarer and rarer at each stroke of the piston, the whole can never be exhausted.

The pump which we have described is called the Single-barrelled, or Smeaton's Air-pump. Hawksbee's Air-pump is fitted with two barrels instead of one (Fig. 29). The two pistons are moved by a toothed wheel in such a manner, that one descends while the other ascends, and thus the air can be forced out with greater rapidity.

62. *To find the density of the air in a receiver after any number of turns of the wheel.*

Let R be the volume of the receiver and pipe, and b the volume of each barrel. Then the air which occupied the space R when H was at the bottom of the barrel, fills the space $R + b$, when H ascends to the top.

If therefore $d =$ the density before the stroke, and d_1 the density afterwards, we shall have

$$d_1 (R + b) = d \cdot R.$$

In the same manner it will be found if d_2 be the density after two turns, d_n the density after n turns, that

$$d_2 (R + b) = d_1 \cdot R \qquad \therefore\ d_2 (R + b)^2 = d \cdot R^2$$
$$d_3 (R + b) = d_2 \cdot R \qquad \therefore\ d_3 (R + b)^3 = d \cdot R^3$$
$$\therefore\ d_n (R + b)^n = d \cdot R^n.$$

Thus it appears that the density of the air decreases in a geometrical progression.

The Siphon Gauge.

This is a glass tube s (Fig. 29) screwed to a pipe communicating with the receiver. One end *a* is

Fig. 29.

closed, and a portion of the tube, rather more than *a b*, completely filled with mercury.

If it be not more than 28 inches in length, the tube *a b* will at first remain filled; but as the exhaustion proceeds, the mercury will sink in *a b* and rise in *b c*, and the difference in the heights of the mercury in *a b* and *b c* will always measure the pressure in the receiver.

Thus, if we call the pressure of the air which supports 76 centimetres of mercury *one atmosphere*, and let n centimetres = the difference in the heights, the pressure of the air in the receiver is $\frac{n}{76}$ths of one atmosphere.

If d be the density of the air outside, and d_1 the density in the receiver, since density is proportional to the pressure,

$$\frac{d_1}{d} = \frac{n}{\text{height of barometer.}}$$

The Diving-Bell.

63. The diving-bell is a strong iron vessel, which is immersed in water with its mouth downwards. The air within prevents the water from filling the vessel; but as it descends the pressure increases, and the air is compressed. Let us suppose that the pressure of the air at the surface is equal to the weight of a column of water 33 feet high, then when the bell has descended 33 feet, the air within will be compressed to half its original bulk. The pressure of the air inside would support a column of mercury twice as high as the mercurial barometer at the surface. At a depth of twice 33 feet the air would be compressed to one-third its original bulk, and so on.

To prevent the advance of the water in the bell, and to supply the workmen with fresh air, a tube, connected with a force-pump at the surface, passes to the bell. There is also a tube through which the workmen can allow the foul air to escape by turning on a stop-cock.

The bell is suspended by chains or ropes.

Exercise.—A cylindrical diving-bell 8 feet long descends until the water in the bell is 80 feet below the surface, when the height of a water-barometer is 32 feet. If no air has been supplied from above, to what height has the water risen in the bell?

The pressure at the surface = the pressure of 32 ft. of water.

" " a depth of 80 ft. = the pressure of 112 ft. of water.

With a pressure of 32 ft. of water, the air occupies 8 ft. of the cylinder.

∴ With a pressure of 1 ft. of water, the air would occupy 8×32 ft.;
and with a pressure of 112 ft. of water,

$$\frac{8 \times 32}{112}, \text{ or } 2\tfrac{2}{7} \text{ ft.}$$

Exercises.

1. When the height of the barometer is 76 centimetres, find the height to which water will rise in a tube from which the air has been withdrawn, the specific gravity of mercury being 13·596. —*Ans.* 1033 centimetres.

2. What would be the height of the barometer if a liquid of specific gravity 2·5 were used, when the mercurial barometer stands at 76 centimetres?—*Ans.* 413·3 centimetres.

3. When the water barometer stands at 33 feet, what would be

the height if alcohol of specific gravity ·825 were used instead of water?—*Ans.* 40 feet.

4. What would be the effect of making a small aperture at the highest point of a siphon?

5. What difference is made in the height of the column of mercury in a barometer when the tube is inclined?

6. How would the action of a siphon be affected by carrying it up a mountain?

7. If when the barometer stands at 30 inches, the air in the glass cylinder represented in Fig. 28 is half exhausted; find the pressure on a square inch, taking the specific gravity of mercury as 13·5, and the weight of a cubic foot of water as 1000 ozs.

8. If the weight of a cubic inch of mercury be 7·8 ozs., what is the pressure of the air on a square inch when the mercury stands at 29·5 inches?—*Ans.* 230·1 ozs.

9. What would be the height of the water barometer in the above case, the specific gravity of mercury being 13·5?—*Ans.* 398·25 inches, or 33 feet $2\frac{1}{4}$ inches.

10. A barometer is partly filled with water and partly with mercury, and the height of the water is three times that of the mercury; find the height when the pressure of the atmosphere is 1020 grammes on a square centimetre, the specific gravity of mercury being 13·596.

Solution. Let x be the height of the mercury;

then $3x =$ the height of the water,

and $13{\cdot}596\,x =$ the height of water equal to x centims. of mercury.

Now, a water-barometer would stand at 1020 centimetres.

$$\therefore\ 3x + 13{\cdot}596x = 1020$$

$$\therefore\ x = 61{\cdot}5 \text{ centimetres.}$$

11. How will a fall in the barometer affect the action of a siphon?

12. How will a change in the barometer affect the pressure on the base and sides of a cubical vessel filled with water?

13. Find the greatest height over which a liquid of specific gravity 3·5 can be carried by means of a siphon, when the barometer stands at 760 millimetres.

14. If a vessel of water containing a floating body be placed under the receiver of an air-pump, and the air gradually exhausted, what will be the effect on the floating body?

Solution. The weight of the floating body is equal to the weight

of fluid (air and water) displaced; hence, as the surrounding air is withdrawn—that is to say, as the air displaced by the body is diminished—the water displaced must increase. When the receiver is completely exhausted, the body will sink, until the weight of the water displaced is alone equal to the weight of the body.

15. To what height would the mercury rise in a Torricellian tube, if it be immersed in water until the surface of the mercury in the cistern is 45 inches below the surface of the water, supposing the barometer to stand at 30 inches, and the specific gravity of mercury to be 13·5.—*Ans.* 33⅓ inches.

16. The receiver of an air-pump is five times as large as the barrel; after how many strokes will the density of the air be diminished by nearly a half?—*Ans.* 3.

17. In the above example, how much air has been removed after six strokes?—*Ans.* Taking the original volume as unity, the air removed is

$$1-\left(\frac{5}{6}\right)^6.$$

18. What would be the effect of making a small hole in the top of a diving-bell?

19. What is the pressure on a square centimetre in a diving bell, the surface of the water in the bell being 12 metres below the surface above, and the mercurial barometer at the surface standing at 76 centimetres (specific gravity of mercury = 13·596). *Ans.* 2·233296 kilogs.

20. If a block of wood be floating on the surface of the water within a diving-bell, how will it be affected by the descent of the bell?—(See question 14 above.)

PART II.

VIII.—PRESSURE OF FLUIDS.

64. *Definition.—A fluid is a collection of material particles which can be displaced by the slightest force.*

65. The pressure of a fluid on a plane when uniform is measured by the force exerted on a unit of area. When we speak of the *pressure at a point* of the fluid we mean the pressure on a square unit containing that point; for example, when we say that the pressure at a depth of 30 feet below the surface of a lake in 13 lbs. on the square inch, we mean that a horizontal plane 1 inch in area at a depth of 30 feet will sustain a pressure of 13 lbs.

If the pressure be variable over the plane, as, for instance, on the sides of a vessel filled with fluid, the pressure at any point is the pressure which would be exerted on a unit of area, if the pressure over the whole unit were exerted at the same rate.

When the pressure is uniform over an area containing a square units, if P be the whole pressure, the pressure on one square unit, that is the pressure at any point in the area $=\frac{P}{a}$

When the pressure is variable if a be the area of

a very small plane at a point A, and P the pressure on this plane, then the limit of the ratio $\frac{P}{a}$ as a (and consequently P) is diminished indefinitely, is the pressure at the point A. This is the same as saying that if a be very small, we may suppose the pressure on it uniform, and therefore the pressure at a point in it $=\frac{P}{a}$.

66. In order to apply the principles of statics to the case of fluids in equilibrium, the following assumption is frequently employed.

Any portion of a fluid at rest may be supposed to become solid without affecting the conditions of equilibrium.

The Pressure at any Point of a Fluid at Rest is the same in all Directions.

67. Suppose a small triangular prism of the fluid, having the ends and one face vertical and another face horizontal to become solid. Let A B C be a vertical section of the prism bisecting its length. Taking the length of the prism for the unit let B C $=a$, A C $=b$, A B $=c$. Since the faces are very small the pressures on them may be considered uniform; hence, if the pressures at points in B C, A C, A B be respectively p_1, p_2, and p_3, the pressures on these faces will be respectively p_1a, p_2b, p_3c. The forces which act on the prism are its weight and the pressures on its sides, all of which are in the plane

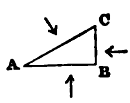

Fig. 30.

A B C, and the pressure on the ends, which are perpendicular to all the other forces, and therefore balance each other. From the equilibrium of the forces in the plane A B C, namely:—

p_3c—the weight perpendicular to A B.
p_1a „ B C.
p_2b „ A C.

By the triangle of forces we have

$$\frac{p_1a}{p_2b}=\frac{\mathrm{B\,C}}{\mathrm{A\,C}}=\frac{a}{b}$$

hence $p_1=p_2$, whatever the inclination of A C, or the size of A B and B C. By supposing these lines indefinitely diminished, p_1 and p_2 become pressures in different directions at the point B.

Any Pressure applied at a point in a Fluid is transmitted equally to all other points.

68. Let A and B be two points, and suppose a thin square prism of the fluid connecting them to become solid. The forces acting on the prism are in equilibrium; hence, if a pressure P be applied at A, an equal pressure must be applied at B to balance it; in other words, the pressure applied at A is transmitted to B.

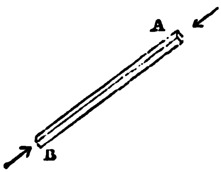

Fig. 31.

The Pressure upon any plane by a fluid at rest must be in a direction perpendicular to the Plane.

69. The pressure of the fluid is in equilibrium with the resistance of the surface, which we know to

act perpendicular to the plane, consequently this must also be the direction of the fluid pressure.

The Pressure of a Liquid at rest is the same at all points of the same Horizontal Plane.

70. Let A and B be points in a horizontal line A B. Take a small square, having its centre at A in a vertical plane, and draw horizontal lines through the angles.

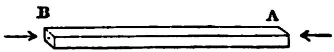

Fig. 32.

A vertical plane through B will cut these lines in the angular points of a second square equal to that at A. Imagine the square prism A B to become solid; it is kept at rest by two sets of forces, namely, one consisting of the fluid pressures on the faces, and the weight, all of which act in a direction perpendicular to A B, and the other consisting of the fluid pressures P P_1 on the two ends. Now by a principle of statics (§ 34), each of these sets must be in equilibrium amongst themselves, hence $P=P_1$.

If the vertical areas are taken very small, we may suppose the pressure uniform, and if a be the area of the square, the pressure at $A=\frac{P}{a}$, and that at $B=\frac{P_1}{a}$, and we have seen that these are equal.

To find the Pressure at any given depth in a heavy homogeneous liquid at rest.

71. Let D be a point in the fluid at a given depth, draw D M vertically to the surface.

Describe a thin cylinder about D M, having its

base horizontal, and imagine it to become solid.

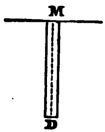

Fig. 33.

Then the solid body D M is kept at rest by the fluid pressure on the end D, which is vertical; the weight of the solid also vertical, and the fluid pressures on the curved surfaces which are all horizontal. Hence the fluid pressure on P must be equal to the weight, and therefore if a be the area of the base, w the weight of a unit of volume, p the pressure at P and d the depth M D

$$p\,a = w\,a\,d$$

$$\text{or}\quad p = w\,d$$

that is, the pressure at any depth varies as the depth below the surface.

Similarly, if two points be taken in the same vertical line at depths d and d', and if p and p' be the pressures at these points,

$$p'a - p\,a = w\,a\,d' - w\,a\,d$$

$$\text{or}\quad p' - p = w\,(d' - d)$$

that is, the difference of the pressures at any two points varies as the vertical distance between the points.

72. That the pressure is proportional to the depth is also true when the points considered are not vertically below the surface, as in the figure.

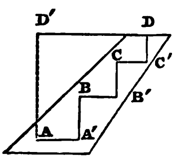

Fig. 34.

The pressure at a point A is the same as at A' in the same horizontal line. Draw A' B vertical, and if it does not meet the surface draw B B' horizontal, and so on, until a vertical line C'D is drawn meeting the surface.

The pressure at C′ is due to the depth C′ D, and it is equal to the pressure at C in the same horizontal line. The pressure at B is the pressure at C, together with the pressure due to the height B′ C, that is, to the pressure due to the height (B′ C + C′ D). By continuing the process we find that the pressure at A is that due to the depth (A′ B + B′ C + C′ D), that is, to the depth of the pcint A below the surface.

The Surface of a fluid at rest is a Horizontal Plane.

73. Take two points in the liquid in the same horizontal plane H, at depths d and d'. The pressures at these points are proportional to $d\, d'$ by § 71, and they are equal by § 70, hence $d = d'$.

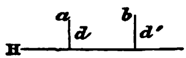

Fig. 35.

Thus the points a and b on the surface are at same distance from the plane H, and therefore lie in a plane parallel to H, that is, in a horizontal plane.

The common Surface of two Liquids that do not mix is a Horizontal Plane.

74. When two liquids do not mix, the pressure at a given depth of the lower is the pressure due to the depth below its surface, added to the pressure at the surface. If w_1 be the weight of a unit of the first fluid, and d its depth, $d\, w_1$ is the pressure on a unit of the surface of the second fluid. If w_2 be the weight of a unit of the latter, and x the depth below

the surface of a given point P in it, then the pressure at P $= w_2 x + w_1 d$.

Similarly, the pressure at any other point will be $w_1 d_1 + w_2 x_1$.

Take these points in the same horizontal plane, then the pressures are equal, and the depths of the points below the first surface are equal,

hence $w_1 d + w_2 x = w_1 d_1 + w_2 x_1$,

and $d + x = d_1 + x_1$,

by multiplying the second equation by w_1, and subtracting the product from the first, we have $x = x_1$.

Pressure on a Horizontal Plane.

75. *The pressure of a liquid on a horizontal plane is equal to the weight of a column of the liquid, which has for its base the area of the plane, and for its height the depth of the plane below the surface.*

Let the figure represent a section of the vessel, and let A B be the area of the plane; draw through the boundary of A B vertical lines to the surface, and suppose the fluid contained within these lines to become solid.

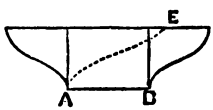

Fig. 36.

The mass thus marked out is in equilibrium, under the action of the horizontal pressures of the surrounding fluid, the vertical re-action of the plane A B and the weight, which is also vertical. But when the forces acting on a body at rest consist of two sets at right angles, each set must be in equi-

librium; hence, the pressure on the base must be equal to the weight of the column.

If the side of the vessel had the position of the dotted line E A, the reaction of this surface on the fluid would be the same as the pressure of the fluid D, which is above it; hence, the introduction of this surface will not alter the pressure on the base.

The whole Pressure on a Surface.

76. The pressure on any surface immersed in a fluid may be found by supposing the surface divided into areas so small that they may be considered planes acted on by uniform pressures.

Definitions.—The sum of the pressures of the fluid on the small plane areas into which any surface may be divided, is termed the *whole pressure* of the fluid on the surface.

The resultant of the fluid pressures on the small plane areas into which any surface may be divided, is termed the *resultant pressure* of the fluid on the surface.

When the surface is a plane, the pressure on the elementary areas are parallel, and their resultant is equal to their sum; hence, in this case the whole pressure is equal to the resultant pressure.

If the surface be curved, it is evident that the resultant pressure will be less than the whole pressure.

The point in which the line of action of the resultant pressure meets the surface is termed the *centre* of pressure.

To find the whole Pressure on a Plane Area in the form of a Rectangle just immersed in a Liquid.

77. Let M N P Q be the rectangle, having the sides M N. P Q horizontal. Draw P R Q S perpendicular to the surface. Suppose the prism to become solid. Let A B C be a section of the prism by a vertical plane through A and C, the middle points of M N and Q P.

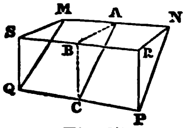

Fig. 37.

The solid prism is under the action of the resultant pressures on the ends, which are equal and opposite, and therefore neutralize one another, and the folowing pressures in the plane A B C—the resultant P_1 of the pressures on the rectangle S P acting perpendicularly to B C, the resultant P_2 of the pressures on the rectangle M P perpendicular to A C, and the weight of the prism, which is vertical.

Thus the prism may be regarded as a solid at rest on an inclined plane under the action of a horizontal force; hence (Statics, p. 8),

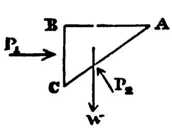

Fig. 38.

$$\frac{P_1}{W} = \frac{\text{height}}{\text{base}} = \frac{BC}{AB}$$

$$\frac{P_2}{W} = \frac{\text{length}}{\text{base}} = \frac{AC}{AB}$$

Let $MN = l$, $AC = b$, $BC = a$, $AB = c$.

The volume of the prism is $\frac{1}{2}AB \,.\, BC \times MN$, or $\frac{1}{2}a\,c\,l$, and if w be the weight of a unit of the fluid, the weight of the prism $= \frac{1}{2}a\,c\,l\,w$;

hence $\quad P_1 = \frac{1}{2}\,a\,clw \times \frac{BC}{AB} = \frac{1}{2}a\,(a\,l)\cdot w.$

Now, $a\,l$=area S R P Q, and $\frac{1}{2}a$=the depth of its C. G.; hence

P_1=the weight of a column of fluid having the rectangle as base, and the depth of its C. G. below the surface for height.

Similarly, $$P_2 = W \cdot \frac{AC}{AB}$$
$$= \tfrac{1}{2}aclw \times \frac{b}{c}$$
$$= \tfrac{1}{2}a(bl)w.$$

$b\,l$ is the area of the rectangle M N P Q, and $\frac{1}{2}a$ the depth of its C. G.

We see, therefore, that if a rectangle immersed in a fluid be turned about its C. G. in any direction, the pressure on it remains the same, and is equal to the weight of the column above it when it is horizontal.

The whole Pressure on any Area.

78. The above is a particular case of the general proposition we are about to demonstrate.

The whole pressure of a liquid on any surface is equal to the weight of a column of liquid, the area of whose base is equal to that of the surface, and whose height is equal to the depth of the centre of gravity of the surface.

Let the surface be divided into a great number of very small rectangular areas a_1, a_2, a_3.........and let the depths of the centres of gravity of these areas be d_1, d_2, d_3.

The pressures on them will be

$$w\,a_1d_1 \quad w\,a_2d_2 \quad w\,a_3d_3 \ldots\ldots\ldots$$

and therefore the whole pressure is

$$w\,(a_1d_1 + a_2d_2 + \text{etc.})$$

Now, let D be the depth of the C. G. of the surface, and let A be its area, then, by a proposition in Statics,

$$A \cdot D = a_1d_1 + a_2d_2, \text{ etc.}$$

Hence the whole pressure $= w \cdot A \cdot D$.

Let us take a horizontal area M equal to A (Fig. 39), and raise on it a column of height D, then the weight of this column of fluid would be equal to $w \cdot A \cdot D$, that is to the whole pressure on the surface.

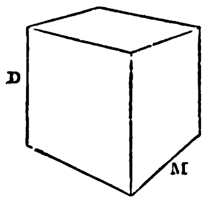

Fig. 39.

79. It must be remembered that D is the depth of the C. G. of the *surface* against which the pressure acts. For example, find the whole pressure on the surface of a hemispherical bowl filled with water, the radius being 3·5 feet.

The area of the surface of a sphere is

$$4 \times \frac{22}{7} \times (\text{radius})^2.$$

Hence the area of the surface of the hemisphere is

$$2 \times \frac{22}{7} \times (3{\cdot}5)^2 = 77 \text{ cubic feet.}$$

The centre of gravity of the surface of a hemisphere is distant from the centre half the radius.

Hence the pressure $= 77 \times 1{\cdot}75 \times 1000$ ozs. $=$ 134750 ozs.

Resultant Vertical Pressure.

80. It is frequently convenient to consider the amount of the pressure of a fluid on a surface in a given direction. The resultant vertical pressure of a liquid on any surface is the weight of the superincumbent liquid.

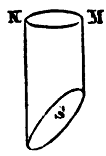

Fig. 40.

Let S be the surface in contact with a fluid at rest. Through the perimeter of S draw vertical lines to the surface M N, thus inclosing a mass of the liquid.

The pressure of the surrounding liquid on this mass is entirely horizontal, consequently the weight of the mass is entirely supported by the reaction of the surface S.

The vertical component of this reaction must therefore be equal to the weight of the mass lying above the area S.

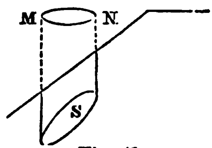

Fig. 41.

By the previous chapter this is true, whether the curve M N be really in the liquid, or only in the horizontal plane through the highest point of the liquid, as in the figure (Fig. 41). Hence it follows that the resultant vertical pressure is the weight of the superincumbent liquid, and its line of action is the vertical through the centre of gravity of the superincumbent liquid.

To find the resultant horizontal pressure in a given direction of a fluid on any surface.

81. Let V V be a fixed vertical plane perpen-

dicular to the given direction, and let horizontal lines through the circumference of the surface S meet the vertical plane in the curve M N. Suppose the liquid thus enclosed to become solid. The solid body M S is in equilibrium under the action of its own weight, the fluid pressures on its curved surface, which are all parallel to the vertical plane, and the fluid pressures on the surfaces M N and S. Hence the horizontal component of the reaction of S must be equal to the pressure on M N, that is to the weight of a column of the liquid, having M N for base, and the depth of the C. G. of M N for height. The line of action will be the horizontal line through the centre of pressure of M N.

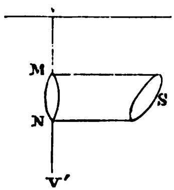

Fig. 42.

82. From these propositions it follows that the centre of pressure of any area immersed in a fluid is vertically below the C. G. of the superincumbent mass.

83. The centre of pressure of an area M N in a vertical plane is in the same horizontal line as the centre of pressure of the inclined area S, of which M N is the projection.

Centre of Pressure.

84. *To find the centre of pressure of any parallelogram having a side in the surface.*

Let A B C be the vertical plane through the middle points A and C of the two horizontal sides. The C. G.

of the prism of fluid above the parallelogram coincides with G, the C. G. of the triangle A B C. The vertical G O through G is parallel to B C, and therefore divides all lines from A to B C in the same proportion; but since it divides the

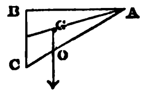

Fig. 43.

line A E through G so that $A\ G = \frac{2}{3} A\ E$ it divides A C, so that $A\ O = \frac{2}{3} A\ C$.

By § 82 this is true also when the parallelogram is vertical.

85. *To find the centre of pressure on a triangle immersed in a fluid, with its base in the surface.*

Let M N C be the triangle. The superincumbent mass is a triangular pyramid, the C. G. of which is found by joining g the C. G. of the triangle with B, and taking G, so that $g\ G = \frac{1}{4} g\ B$. The vertical through G is parallel to B C, and

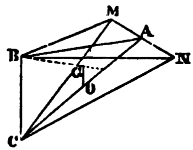

Fig. 44.

divides C g in the same proportion as B g; hence

$$C\ O = \frac{3}{4} C\ g = \frac{3}{4} \text{ of } \frac{2}{3} \text{ of } C\ A = \frac{1}{2} C\ A.$$

By § 82 this is also true when the triangle is vertical.

86. *To find the centre of pressure of a triangle immersed in a fluid, with its base horizontal and its apex in the surface.*

Let A M N be the triangle. The superincumbent

fluid is a pyramid with rectangular base, and its C. G. is found by joining g the C. G. of the base with

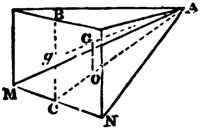

Fig. 45.

A and taking $g\,G=\frac{1}{4}\,g\,A$. Let B C be the vertical through g, and G O the vertical through G, then G O is parallel to g C and $g\,G : g\,A :: C\,O : C\,A$; consequently $C\,O=\frac{1}{4}\,C\,A$.

87. *To find the centre of pressure of a rectangle totally immersed in a fluid, with two sides horizontal at a depth below the surface.*

Let P Q R S be the rectangle, and $p\,q\,r\,s$ its projection by vertical lines on the surface.

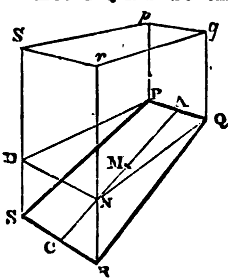

Fig. 46.

The superincumbent mass may be divided into a rectangular prism D E, the C.G. of which is over the middle point M of A C, and a triangular prism D Q, the C. G. of which is over N, such that $C\,N=\frac{1}{3}\,C\,A$.

Let a=area $p\,q\,r\,s$, let $d_1 = q\,Q$, $d_2+d_1 = r\,R$; the weight of the rectangular prism$=w\,a\,d_1$,

„ „ triangular prism $=\frac{1}{2}\,w\,a\,d_2$

hence their ratio is $2\,d_1 : d_2$

The resultant of $2\,d_1$ at M and d_2 at N

is $(2\,d_1 + d_2)$ at P, such that $d_2 \mathrm{CN} + 2d_1 \,.\, \mathrm{CM} = (2\,d_1 + d_2)\,\mathrm{C\,P}$.

Let $\mathrm{A\,C} = l$.

$$\therefore\ d_2 \times \frac{l}{3} + 2\,d_1 \times \frac{l}{2} = (2\,d_1 + d_2)\,\mathrm{CP}.$$

$$\therefore\ \mathrm{CP} = \frac{l\,(d_2 + 3d_1)}{3\,(2d_1 + d_2)}.$$

The depth of P will therefore be the whole depth rR, less the same portion of RN as CP is of CA ; hence

$$\text{the depth} = d_1 + d_2 - \frac{d_2(d_2 + 3d_1)}{3\,(2\,d_1 + d_2)} = \frac{6\,d_1^2 + 6\,d_1 d_2 + 2\,d^2}{3\,(2\,d_1 + d_2)}$$

88. In a similar manner we may find the centre of pressure of a triangle, totally immersed at a distance below the surface, with base horizontal.

The superincumbent fluid may be divided into a square pyramid (Fig. 45), and a triangular prism above it.

Floating Bodies.

89. *To find the resultant pressure on a body immersed in a fluid.*

Imagine the space occupied by the body to be filled with fluid, and suppose this fluid to become solid. The solid mass would be supported by the fluid pressures on its surface; hence the resultant of these pressures must be equal to the weight of the mass, and must act vertically upwards through its centre of gravity.

Since the suppositions made do not affect the form or position of the surface, the resultant pressure on the body must be the same, namely, in magnitude equal to the weight of the fluid displaced by the body,

and in direction vertically upwards through the C. G of the body.

90. In order that a body may float, it is evident, therefore, that it must be lighter than its own bulk of water.

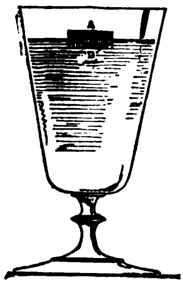

Fig. 47.

A solid A B (Fig. 47) sinks until the weight of water which would fill the space occupied by the immersed part B is equal to the weight of the whole body A B.

The resultant fluid pressure acts always through the C. G. of the part immersed, and the resultant weight acts through the C. G. of the whole body. Hence the conditions that a floating body may be in equilibrium are :—

1°. The weight of the displaced liquid must be equal to the weight of the body.

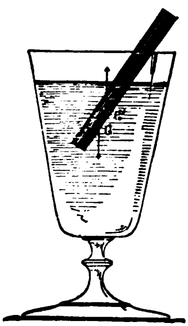

Fig. 48.

2°. The centre of buoyancy and the centre of gravity of the whole body must be in the same vertical line.

If the first condition be not fulfilled, the body will rise or sink. If the second be not satisfied, the body will roll. Thus, suppose the body to be a short rod of wood, loaded at one end with lead, so that the C. G. of the rod is at G near the

lower end, and let it be placed in water. Let G′ be the C. G. of the volume of the rod below the surface, and let G′ be above G. Then, when the rod is disturbed, so that the points G G′ are not in a vertical line, the fluid pressure and the weight form a *couple* which tends to turn the rod about a point between G and G′

The Stability of Equilibrium.

91. The equilibrium is *stable* when the body, on being slightly displaced, returns to its original position.

It is *unstable* if, when slightly displaced, the body moves away from its first position.

If the floating cylinder represented in Fig. 48 were not loaded, and could be made to float with axis vertical, the equilibrium would be unstable. The equilibrium of the loaded rod is stable.

The equilibrium of a sphere is neutral.

The equilibrium is always stable when the C. G. of the floating body is above the C. G. of the displaced liquid. If in this case the body be slightly inclined, the resultant pressure P of the liquid acting at G′, and the weight W of the body acting at G (Fig. 48) tend to bring the body back to its first position.

Although this condition is *sufficient* for stable equilibrium, it is not always *necessary*. It may be that when the body is moved, the water displaced takes a new form, so that the resultant pressure P (Fig. 49) and the weight W act together to make the body return to its first position, although

the C. G. of the body is above that of the liquid displaced.

Let G be the C. G. of the body, and C the C. G. of the part immersed when the body is at rest, V C the line joining them. Let C′ be the C. G. of the part immersed when the body is moved through a very small angle. Let the new vertical through C′ meet V G in M. The position of M for an indefinitely small displacement is termed the *metacentre*. It is evident that the equilibrium is stable when M is above G.

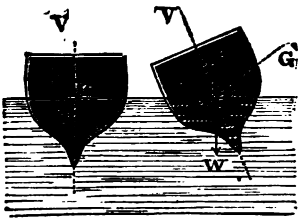

Fig. 49.

Exercises.

1. Find the whole pressure on the rectangle M N P Q (Fig. 37), the area being 8 square feet, and the depth of C 2 feet.—*Ans.* 8000 ozs.

2. Find the pressure on the triangle M N C (Fig. 44), the area being 75 square centimetres and B C being 36 centimetres.—*Ans.* 900 grammes.

3. Find the horizontal pressure on a rectangle 80 centimetres long, and 20 broad, immersed in water, with the shorter side in the surface, and the longer side making an angle of 45° with the surface (Fig. 37)—If AC=80, BC=$40\sqrt{2}$.)—*Ans.* 32,000 grammes.

4. A triangle, the area of which is A, being immersed in a fluid with its angular points at depths h k l below the surface of the fluid; it is required to find the pressure on the triangle.—*Ans.* $\frac{1}{3}$A $(h+k+l)\,w$.

5. Two equal squares are just immersed vertically in a fluid, one with a side, the other with a diagonal vertical; find the ratio of the pressures.—*Ans.* 1 $\sqrt{2}$.

6. A rectangle, whose sides are 20 and 9, is immersed vertically in a fluid, with its shorter side coincident with the surface; divide the rectangle into 4 parts by horizontal lines so that the pressures on each part may be equal.

Solution. The pressure on the whole $=9\times20\times10\times w$

Let x be the breadth of the first part, then $\frac{x}{2}=$ the depth of its C. G., and the pressure on it $=9\times\frac{x^2}{2}w$

$$\therefore 9\times\tfrac{1}{2}x^2 w=\tfrac{1}{4}\times9\times200\,w$$
$$\therefore x=10$$

Let $y=$ the breadth of the next part; the pressure on it $=$

$$9y\left(x+\frac{y}{2}\right)w$$
$$\therefore y\left(x+\frac{y}{2}\right)=\tfrac{1}{4}\times200$$
$$\therefore y^2+20y=100$$
$$y=10(\sqrt{2}-1)=4{\cdot}142$$

Similarly, if $z=$ the breadth of the third $z\,(x+y+\frac{1}{2}z)=50$

$$\therefore z=10(\sqrt{3}-\sqrt{2})=3{\cdot}178$$

The fourth may be found either in the same way $=10(2-\sqrt{3})$, or by subtracting the other three from 20.

7. Divide the above rectangle into five parts by horizontal lines, so that the pressures are equal.

$$\begin{array}{ll} \textit{Ans.}\text{—}4\sqrt{5} & \text{or } 8{\cdot}9443 \\ 4\sqrt{5}\,(\sqrt{2}-1) & \text{or } 3{\cdot}7048 \\ 4\sqrt{5}\,(\sqrt{3}-\sqrt{2}) & \text{or } 2{\cdot}8423 \\ 4\sqrt{5}\,(\sqrt{4}-\sqrt{3}) & \text{or } 2{\cdot}3966 \\ 4\sqrt{5}\,(\sqrt{5}-\sqrt{4}) & \text{or } 2{\cdot}1115 \end{array}$$

8. If a cubical vessel be filled, half with mercury and half with water, compare the pressure on the sides with the pressure on the base, which is horizontal; the specific gravity of mercury being 13·596.—*Ans.* 4·149 : 14·596.

9. A side of the base of a square pyramid is 10 inches, the altitude is 22 inches; if the pyramid be filled with water, compare th pressure B on the base with the pressure S on each side, and with the weight W of the water.—*Ans.* B : S : W : : 3 : 2·256 : 1.

10. If two spheres whose radii are as 3 and 5 be just immersed in a fluid; compare the pressures on them.—*Ans.* 27 : 125.

11. Find the whole pressure on the surface of a solid cone including its base, when immersed in a fluid with its axis vertical and its vertex just at the surface of the fluid.—*Ans.* If $h=$ the height, $r=$ the radius of the base, $l=$ the slant length, $w=$ the weight of a unit of fluid, the pressure $=\frac{1}{3}\pi hr\,(3r+2l)\,w$.

12. A solid cone of metal, the height of which is 12 centimetres, the area of the base 40 square centimetres, and specific gravity 12, completely immersed in liquid is supported by a string; find the tension on the string.—*Ans.* 1760 grammes.

13. A wooden plank, 12 feet long, floats in water, and a weight of 18 lbs. is placed at one end of the plank; find the weight, which placed 4 feet from the other end will keep the plank in a horizontal position.—*Ans.* 54.

14. A small piece of lead is attached to a wooden sphere, and the weight of the sphere is then half that of an equal quantity of water; find its positions of equilibrium in water, and examine the stability of the equilibrium.

15. A cylinder of wood, 4 feet long, floats with its axis vertical in a fluid of twice its specific gravity; if an upward force of 12 ozs. will raise it 8 inches; find the force required to depress it 8 inches.

16. A block of ice, the volume of which is a cubic metre, is observed to float with one twelfth of its volume above the surface, and a small piece of granite is seen imbedded in the ice; find the size of the stone having given the specific gravity of ice $\cdot 9$, and of granite $2\cdot 5$.

17. Divide a cylinder of height h, which is just immersed in a fluid with its axis vertical, into 2 parts, so that the pressures on them may be equal.—*Ans.* By a plane at a depth of $h \div \sqrt{2}$.

18. A cylinder, 15 inches in length, is just immersed vertically in two fluids that do not mix, whose densities are as $1:2$; and the pressures upon the two parts of the convex surface of the cylinder are as $2:3$; find the length of the cylinder immersed in each fluid. —*Ans.* 10 and 5 inches.

19. To the end of a wooden rod a small piece of lead is attached of twice its weight; find the density of a liquid in which it will float at any direction to the vertical, the density of the wood being unity.—*Ans.* $4\frac{1}{2}$.

20. A cylindrical glass tumbler weighs 143 grammes, its external diameter is 7 centimetres, and its height 8 centimetres.

When it floats in water with its axis vertical, how much is immersed, and what additional weight must be placed in it, in order that it may sink.—*Ans.* $3\frac{5}{7}$ centimetres, 165 grammes.

21. A uniform rod floats partly immersed in water, and is supported at one end by a string; prove that if the length immersed remain unaltered, the tension of the string is independent of the inclination of the rod to the vertical.

22. A conical vessel is filled with water, and closed by a lid; compare the pressures on the curved surface when the vertex is upwards, with the pressure on the same surface when the vessel is held with vertex downwards.—*Ans.* 2 : 1.

23. A solid cone and a solid hemisphere which have their bases equal, are united together base to base, and the solid thus formed floats in water with its spherical surface partly immersed; find the height of the cone, in order that the equilibrium may be neutral.

IX.—THE STEAM-ENGINE.

92. Steam-engines may be divided into classes, according to several particulars; for example, engines may have cylinders fixed or oscillating, vertical or horizontal. They may have a condensing apparatus, or no condensing apparatus. We need, however, only recognize the third distinction, and divide engines into two classes—those in which the steam is condensed after leaving the cylinder, commonly called low-pressure, and those in which the steam, after working the piston, passes to the atmosphere, called high-pressure. The exigencies of modern

practice have tended to alter this distinction of low-pressure and high-pressure engines very materially. In former times, condensing engines always worked with low-pressure steam; now they frequently work with steam of high-pressure. Hence the terms con-

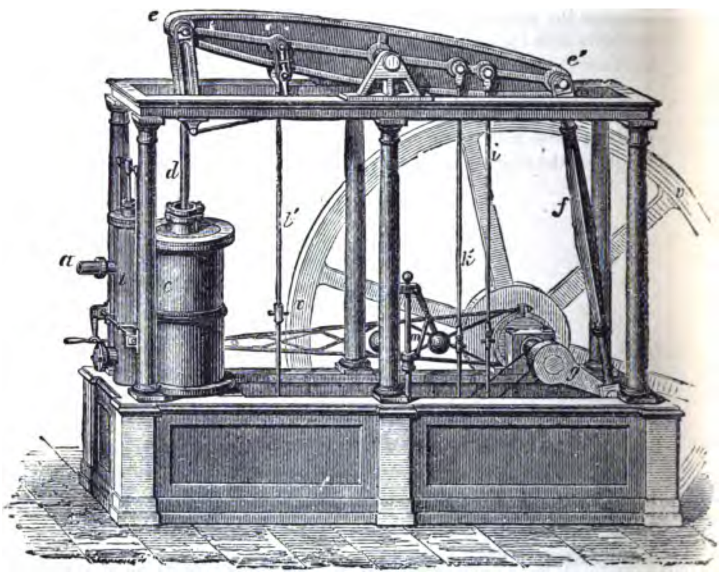

Fig. 50.

densing and non-condensing more accurately define the two classes. The form of engine illustrated as a whole in Fig. 50, and in section in Fig. 51, is termed Watt's steam-engine. The steam from the boiler passes along the steam-pipe *a* to the valve-casing,

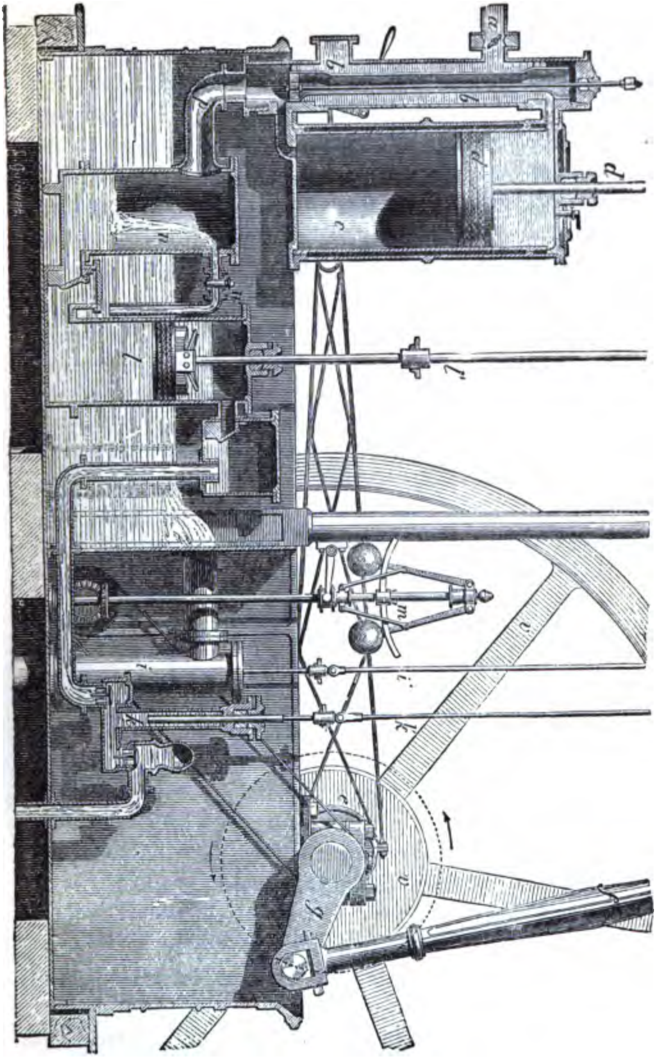

Fig. 51.

from whence it is *distributed*, as it is termed, to the upper and under sides of the piston, producing its alternate up-and-down motion in the cylinder *c*. In the position indicated in the figure, the steam enters the upper part of the cylinder, and forces down the piston. After working the piston, the steam passes by the pipe *p* to the condenser *n*, where it is condensed by coming in contact with a jet of cold water. From the condenser, the water of condensation, together with the air which obtains admission through the steam, and which, if allowed to accumulate, would ultimately prevent the engine working, is drawn off by the air-pump *l*, and delivered to the hot-well. An arrangement of valves prevents the water from returning to the air-pump from the cistern, and also prevents the water which may remain at the bottom of the air-pump from being again forced into the condenser on the down-stroke of the air-pump piston. The condenser and air-pump are placed in a cistern filled with cold water, supplied by the pump. The jet of cold water which plays over the condenser is supplied from the cistern, and is regulated by a stop-valve *n′*.

The piston-rod *d* is connected with the end of the working beam, and is kept parallel by a beautiful arrangement of levers termed Watt's parallel motion. The other end of the beam is joined to the upper end of the connecting-rod *f*, which at its lower end is attached to the crank *g*. To equalize the motion, a heavy wheel, the fly-wheel *v*, is keyed on to the crank-shaft. In the revolution of the crank there are two positions, called the dead points, at both of

which the power of the engine has no effect in causing revolution, when the piston is at the termination of the up-and-down stroke. By the momentum acquired by the fly-wheel while receiving the full power of the engine, the crank is carried past its dead points.

The Slide-Valve.

93. The arrangement by which the steam is alternately led into the upper and lower part of the cylinder is termed a slide-valve. The engine itself regulates the motion of the slide-valve by means of the eccentric *e*. There are several kinds of slide-valves; that represented at *b* in Fig. 51 is termed, from its shape, the long D valve.

The slide-valve represented in Fig. 52 is termed the three-ported valve, because there are three openings L E L′ between which the valve plays. L is called the upper *steam-port;* L′ the lower *steam-port;* E the *exhaust-port.* The steam enters alternately by L and L′, and leaves the cylinder by the opening E. The action of the valve will be better understood by reference to Fig. 53, in which the three-ported valve *t* is connected with the cylinder, the piston being on the point of ascending. The steam passes in the direction of the arrow through the tube *y*, while the exhausted steam is carried into the air by a tube, the

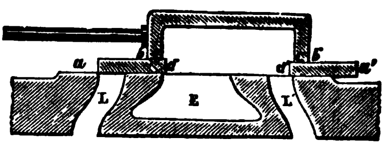

Fig. 52.

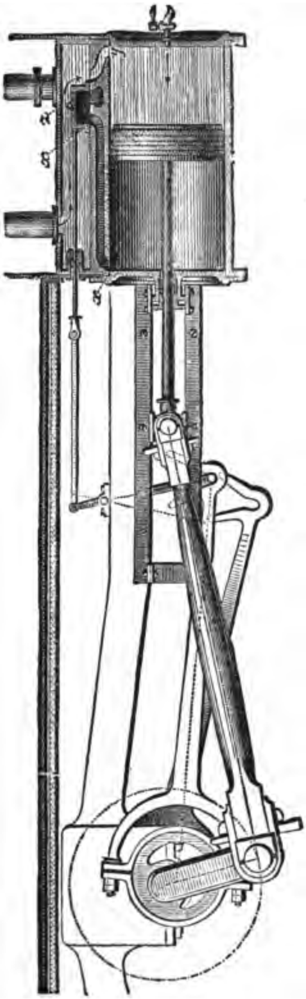

Fig. 53.

end of which is seen at *z*. The steam is not admitted at the top or bottom of the cylinder at the moment when the piston arrives there, but a little before this. This tends to break the force of the shock which would arise in suddenly changing the direction of the motion, in the mass of matter composing the piston, and the parts connected with it. Also, when the piston has been pushed forward a certain distance by the full force of the steam, the supply from the boiler is then usually stopped, and the piston is impelled by the elastic force of the steam already in the cylinder. The engine is then said to work expansively. In some

cases the steam is cut off at a half-stroke, in some at one-third, and in others at a much smaller proportion of the entire stroke. The method of effecting this result will be seen in Fig. 52, where the slide-valve has commenced the return to the left, the compartment of the cylinder on the right is still in communication with the air, while, on account of the length of the foot *a d*, the steam is already cut off from the left.

94. When the steam is cut off at one-third of the stroke, acting *expansively* for the remaining two-thirds, the machine has only half the power it would have if the steam had access to the cylinder during

Fig. 54.

the whole course; thence half the maximum force is obtained at the expense of one-third of the steam.

Sometimes the slide-valve is double, as is represented in Fig. 54, the second valve B B′ being added to regulate the amount of steam used according to the work imposed on the machine.

The second valve can be regulated by means of the screws on the eccentric rods to cut off the steam at any part of the stroke.

The High-Pressure Engine.

95. A non-condensing or high-pressure engine, with horizontal cylinder, is represented in Fig. 55.

The steam from the boiler enters the valve box at A, and in the position indicated presses to the right of the piston B, while the steam in the left end, C of the cylinder is being driven through the pipe E into the air. Before the piston reaches the bottom of the cylinder, the eccentric attached to the lever G causes the slide-valve T T′ to shut off the steam from the upper part of the cylinder, and give a free passage for it to the part C below.

The non-condensing engine is more simple, and consists of fewer parts than that which has just been described. It is generally used for locomotive engines, steam carriages, and steam vessels required to possess lightness and rapidity. Although it is more elementary and simple than the other, it was not invented until many years after the condensing engine had been brought nearly to perfection. In condensing engines the pressure of the steam in the boiler very frequently does not exceed from 4 to 6 lbs. on the square inch; but in the present species, when there is no condenser, and the steam is allowed to pass into the open air, its pressure must necessarily exceed that of the atmosphere, in a sufficient proportion to supply a force equal to the work that is to be done. The pressure is seldom less than 20 lbs. on the square inch. In locomotive engines the pressure is usually from 50 lbs. to 60 lbs. per square inch.

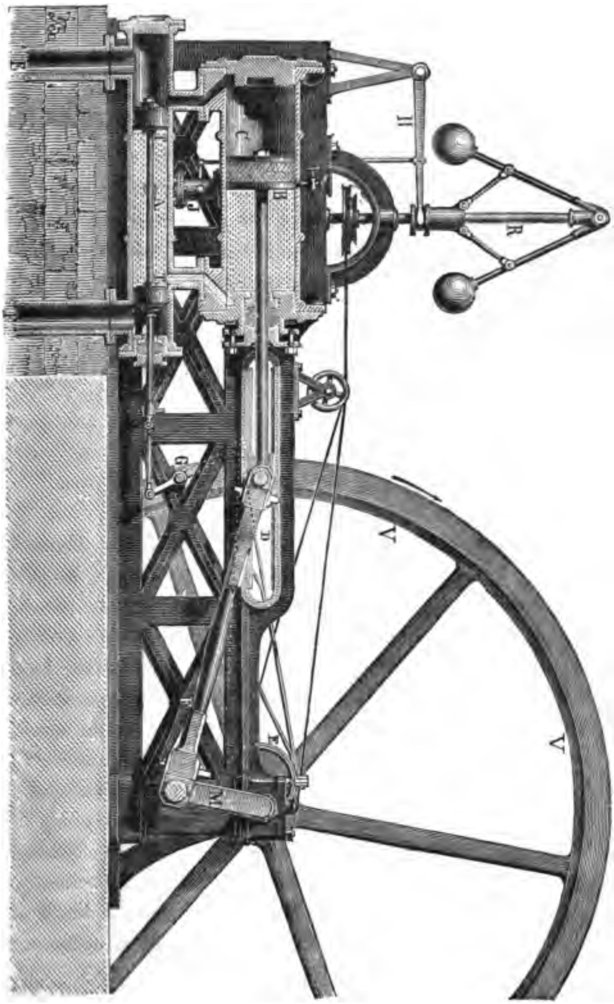

Fig. 55.

The Governor and Throttle-Valve.

96. The supply of steam to the cylinder is regulated by the throttle-valve (*a*, Fig. 50), a circular metal plate, fitting the steam-pipe and moving on a horizontal axis. The edges of the plate are bevelled, so that it is steam-tight when closed. The throttle-valve is connected by a lever with the governor (*m*, Fig. 50, and B, Fig. 55). As the speed of the engine increases the balls fly outward ("Mechanics," p. 195), the lever H is raised, and the valve partially closed.

Locomotive Engine.

The locomotive engine differs from those already described in several important features. Such engines require to be smaller and lighter than stationary engines; hence the apparatus for condensation is rejected and high pressure is used. In the structure of the boiler there is an important peculiarity. It is not one large mass as in the stationary boiler, but an oblong cylinder, through which a number of tubes, usually about ninety, are arranged horizontally in communicating with the furnace and chimney. The heated air passes through these tubes; by this means a very large surface is heated in contact with the water. They are seen in transverse section in Fig. 56, and in longitudinal section in Fig. 57.

It will be seen that the boiler composes the greater part of the engine. It is usually about 3 feet in diameter and 8 feet in length. In front is the smoke-box, having the chimney above and the cylinders lying horizontally below.

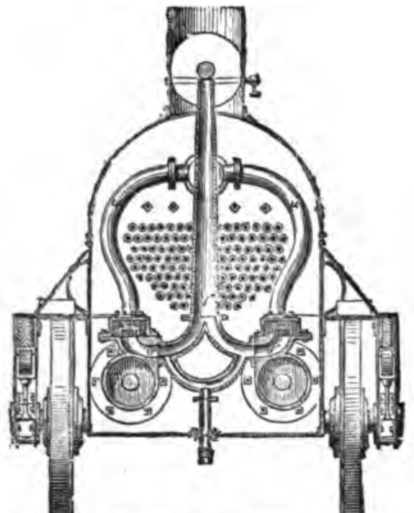

Fig. 56.

At the back of the engine is the fire-box, almost completely surrounded with water. On the top of the engine, proceeding from the smoke-box backwards, we see the man-hole M, by which the boiler can be cleaned or repaired, and the safty-valves V′ V.

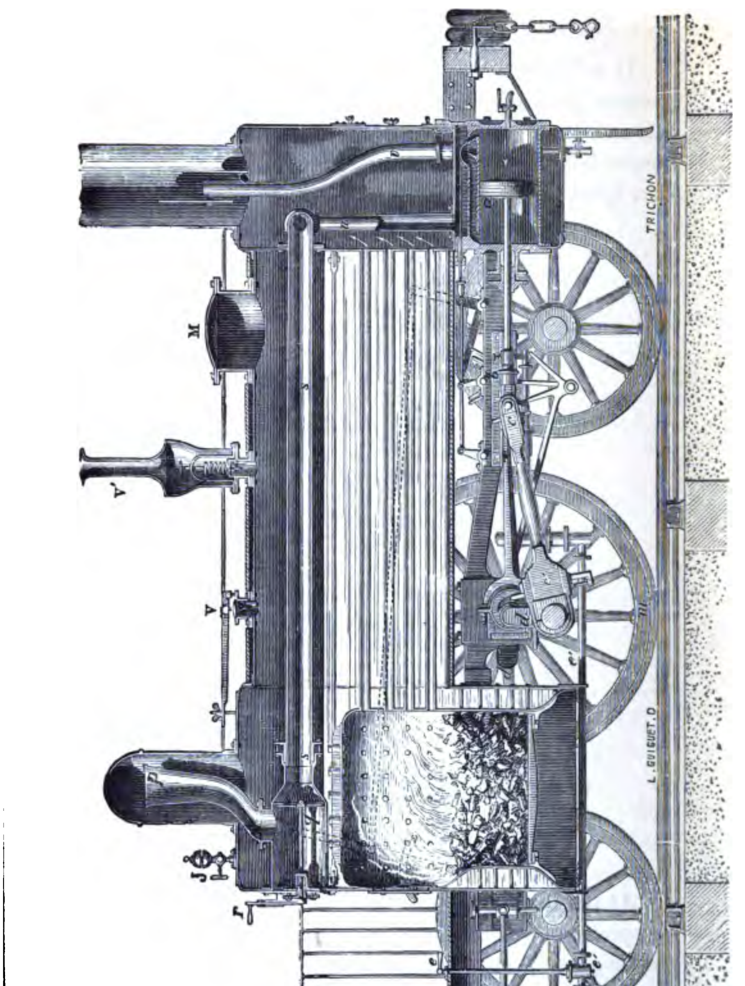

Fig. 57.

The second valve v is under the control of the driver, but the front valve v′ is closed by a strong spring and is free.

The hemispherical chamber in front is termed the *separator*, and is used to collect the steam before it is conveyed by the pipe *p* to the steam-pipe *s*, and thence to the cylinder *a*. The pipe *p* rises high in the separating chamber to prevent the water when agitated by the motion from descending to the cylinders.

A regulator *q* is fixed at the junction of the steam-pipe *s* with the tube *p*, for the purpose of diminishing the flow of steam.

After moving the pistons the steam escapes from the cylinders by two pipes

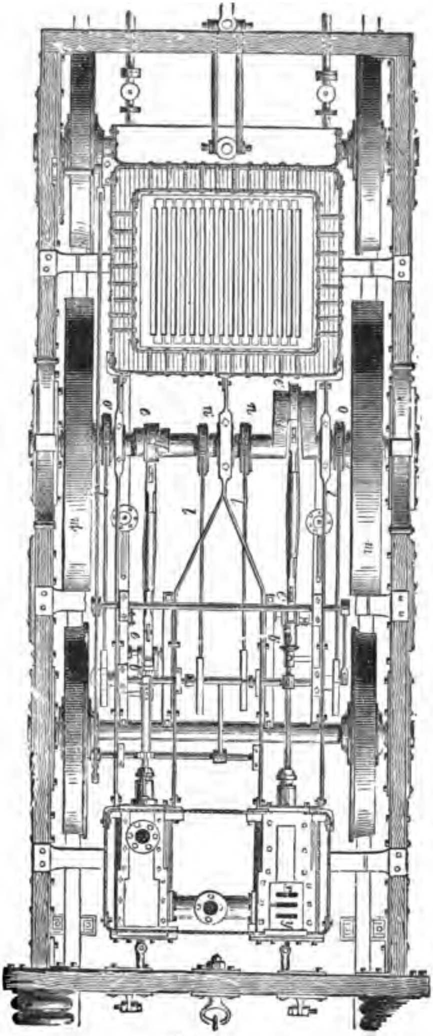

Fig. 58.

meeting in the common tube, or *blast-pipe v*, and thence rushes into the chimney, and increases the draught. When the expedient of turning the exhausted steam into the chimney was first adopted, it was found that the speed of the locomotive on which the experiment was tried was thereby doubled.

Figure 58 represents a horizontal section of the locomotive; *m m′* are the *driving wheels*, *e e′* the cranks, set at right angles so that their *dead points* do not coincide.

The reversing gear consists of two pairs of eccentrics *o*, *o′*, *n*, *n′* fixed to the crank-shaft, one giving the forward, the other the backward motion. These eccentrics are connected with curved links which produce what is called the "link-motion," introduced by Mr. Stephenson.

The Power of Engines.

98. The working power of a steam-engine is estimated in horse power. One-horse power as applied by engineers to the steam-engine is 33,000 foot-pounds per minute ("Mechanics," p. 203.)

In order to calculate the effective power we require to know

(1.) The space through which the piston is moved per minute.

(2.) The size of the piston.

(3.) The mean effective pressure of the steam on the cylinder.

The pressure in the cylinder is found by an instrument devised by Watt, termed an *indicator*. It consists of a small cylinder 8 inches long and about 2 inches in diameter, communicating directly with the cylinder, and supplied with a piston. When the pressure in the cylinder varies, the piston of the indicator rises or falls. A pencil attached to the indicator traces a curve on paper as the piston moves, from which the mean pressure of the steam can be calculated.

Example.—Find the horse power from the following data:—

Length of stroke, 50 inches.

Diameter of piston, 21 inches.

Number of strokes per minute, 30.

Mean effective pressure, 10 lbs. per square inch.

In 1 minute the piston moves through

$$50 \times 2 \times 30 = 3000 \text{ inches} = 250 \text{ ft.}$$

Area of piston $= \left(\frac{21}{2}\right)^2 \times \frac{22}{7} = 346\frac{1}{2}$ square inches.

Mean pressure on whole piston $346\frac{1}{2} \times 10$ lbs.

Therefore the number of foot-pounds

$$= 3465 \times 250.$$

And the horse power $= 3465 \times 250 \div 33000 = 26\frac{1}{4}$.

The power of an engine is frequently estimated by the quantity of steam it can produce in a given time. It is estimated that a cubic inch of water when converted into steam is capable of raising a ton one foot high. Hence 15 cubic inches of water converted into steam per minute would be equivalent to one-horse power.

Relation of Heat expended to Work done by the Steam Engine.

99. In the action of the steam engine the following phenomena occur :—Cold water is heated in the boiler and becomes saturated steam above 100° C.; steam passes into the cylinder and does work by driving the piston; this steam then passes into the condenser or into the air, where it gives up heat and condenses to water.

Now the experiments of Hirn, Joule, and others, have proved that there is a difference between the amount of heat taken up in the boiler and the amount given up by the exhausted steam in the condenser, so that a certain quantity of heat disappears as heat. There is a fixed relation between this difference of the heat taken in and the heat given up, to the work done. If the heat taken in by the water in the boiler during a given time be sufficient to heat Q_1 lbs. of water one degree Fahrenheit, and the heat given out by the exhausted steam in condensing be sufficient to heat Q_2 through one degree, then $Q_1 - Q_2$ multiplied by 772 gives the number of foot-pounds of work done in the same time. This factor, 772, is the dynamical equivalent of heat. If the gramme and the metre are the units of measurement the corresponding number is 423.

The Motion of Liquids.

100. The subject of the motions of fluids has been investigated by many mathematicians, but their investigations on this subject are, as a rule, too elaborate and abstruse to be admitted into the present work. The following practical results are, however, worthy of attention :—

Friction.—1st. The friction of waters in rivers, channels, or conduit pipes increases as the square of the velocity, so that if the velocity be only doubled, the impediment to motion has a four-fold increase.—2nd. The friction against the sides of pipes increases nearly in the inverse ratio of the square roots of their lengths.—3rd. The friction increasing as above with the velocity, at length becomes an exact balance for any acceleration the fluid might otherwise have, so as finally to produce a completely uniform motion.

Flow through an orifice.—The rapidity of the flow depends not only on the *head* of water above the orifice, but, also, on the shape and size of the orifice; for example, a vessel or reservoir having a very thin base, with a smooth circular hole formed in it, might be supposed more capable of parting rapidly with its water than another vessel having a pipe for it to pass through, because very little friction could be produced; but it is found by experiment that a vessel having such an orifice does not discharge its water so rapidly as another containing the same head of water and area of aperture, but to which a short pipe is attached, and

when the length of pipe is equal to twice its diameter, it produces the most rapid discharge.

The reason why there is this difference is that cross currents of the moving fluid are produced by the conducting pipes, so that to a different extent, in different positions, they oppose or divert the current that otherwise would be formed in the fluid. It is found that if the orifice be supplied with a short delivery pipe, the mouth of which is bell or trumpet shaped, having the form of a natural jet of flowing water, and a length equal to twice the diameter of the narrowest part, the maximum quantity of water is delivered through it in a given time.

Torricelli's Theorem.—The velocity with which a liquid issues from a small aperture in the side of a vessel is the same as would be acquired by a body falling freely from the surface level of the liquid to the centre of the orifice.

In other words, if v be the velocity of efflux, and d the depth of the orifice below the surface level of the fluid, then $v^2 = 2\,gd$.

Proof.—The pressure p per square unit at the orifice is $p = w\ d$ where w is the weight of a cubic unit of fluid (*see* p. 68).

Now, suppose a unit of area at right angles to the flow to move through a small distance x with its velocity v, then the mass of liquid moved out of this space (of volume $= 1 \times x$) is $\dfrac{w\,x}{g}$.

The work done by pressure p in acting through the distance x is px or wdx.

The work accumulated in mass m which acquires a velocity v is $\frac{1}{2} v^2 m$. And by the law of the conservation of energy the work done is equal to the work accumulated.

$$\therefore w\,d\,x = \frac{1}{2} v^2 \frac{w\,x}{g}$$

$$\therefore v^2 = 2\,g\,d.$$

The experimental proof of this theorem may be described as follows: If the aperture is turned upwards (see t, Fig. 8) the particles of liquid will rise to the level of the surface in the vessel. Practically, the resistance of the air and friction tend to reduce the height, and the falling fluid hinders that which is rising. By sloping the jet a little this last hindrance is removed and the jet rises to within one-tenth of the height between the surface and orifice.

Again, if a cylindrical vessel have its sides pierced at different heights by apertures of the same size and shape, and be kept full of water, by catching and weighing or measuring the water that escapes in a given time, the rates of efflux can be compared. It will be found that if any given quantity of water issues in a certain time from one of the holes, double that quantity will issue from another hole of precisely the same form and dimensions if it is situated four times as deep as the first below the surface of the fluid, and a similar hole nine times as deep will deliver three times as much fluid in the same time. The discharge is,

therefore, as the square root of the depth beneath the surface of the fluid.

As the jets are thrown out with a constant velocity, and fall under the action of gravity, they form parabolas. If the apertures are bored horizontally, or at right angles to the sides of the vessel, each aperture will be the highest point of the parabola.

An orifice in the middle of the column, or half-way between the surface and base of the fluid, will throw the water to the greatest horizontal distance; and orifices made at equal distances above and below this middle one will throw their jets to equal horizontal distances on a line level with the base of the vessel.

Vena contracta.—If the particles of water pass through an orifice with a velocity corresponding to the depth, the water flowing out in a second would form a cylinder whose base is the orifice, and whose height is the distance which a particle endued with that velocity passes through in a second. If the area of the orifice is a, the quantity which flows out is $v\,a$. It is found, however, that the quantity of water which flows out in a given time does not exceed three-fifths of the calculated quantity, and, moreover, that the jet contracts at a short distance from the aperture. This contraction is called the *contractio venæ*, and the section of the jet at the point of greatest contraction is about two-thirds of the section of the orifice. The explanation appears to be that all the molecules near the edges of the orifice in endeavouring to pass out exert a pressure in different directions across the jet.

The *vena contracta* diminishes the quantity of flow just as narrowing the aperture would do, and approximately the quantity can be calculated by substituting the contracted area for that of the orifice and applying Torricelli's theorem.

MISCELLANEOUS PROBLEMS.

To find the height to which water rises in a diving-bell, at a given depth, when no additional air is introduced, supposing the bell to have a uniform sectional area inside :—

Let z be the depth of the top of the bell below the surface of the water. Let b be the height of the bell, and H the atmospheric pressure at the surface of the water measured by a water barometer.

Also let x be that portion of the height of the bell occupied by the air at the depth z. The pressure on the air within the bell is equivalent to the weight of a column of water, the height of which is $H + z + x$. Hence, by Boyle's law (p. 49) :—

$$\frac{x}{b} = \frac{H}{H + z + x} \text{ or, } x^2 + x\,(H + z) = H\,b$$

a quadratic equation, the positive solution of which gives the height required.

To find the volume of air at the ordinary atmospheric pressure that must be introduced into the bell at a given depth, to prevent any water from entering :—

Let z be the depth of the top of the bell, and b the height of the bell, as before. Then, if V be the volume of the air in the bell at the normal pressure H, and V'

the volume which the compressed air in the bell at depth $z+b$ would occupy at pressure H, it follows from Boyle's law that

$$\frac{V'}{V} = \frac{H+z+b}{H}$$

$$\therefore V' - V = \frac{z+b}{H} \cdot V$$

in other words, the volume of air introduced, at the ordinary atmospheric pressure, is $\frac{z+b}{H} \cdot V$.

To find an expression for the depth of the centre of pressure of a plane area.

Let the area be divided by horizontal planes into small portions.

Let a_1, a_2, etc., be the areas of these portions.
x_1, x_2, ,, the depths of their C.G.'s.

The pressure on the first portion $= a_1x_1w$.

Similarly the pressures on the others are

$$a_2x_2w,\ a_3x_3w, \text{ etc.}$$

Now, by a principle of statics, if each of these forces be multiplied by the distance of its point of application from the surface, and the sum of the products be divided by the sum of the forces, the quotient will be the depth X of the centre of the forces.

$$\therefore \mathrm{X} = \frac{a_1x_1^2w + a_2x_2^2w + \ldots\text{etc.}}{a_1x_1\,w + a_2x_2\,w + \ldots\text{etc.}}$$

Write $\Sigma\, ax^2$ for the sum of all the terms of which ax^2 is the type.

$$\therefore \mathrm{X} = \frac{\Sigma\, ax^2}{\Sigma\, ax}$$

To find the resultant pressure on the Magdeburg hemispheres.

Place the hemispheres with axis vertical, and imagine the surface to be cut by horizontal planes, and vertical lines to be drawn from the lines of section. We shall then have a number of cylinders with common axis, forming a series of steps. The pressures exerted on the vertical faces of these steps do not oppose the separation of the hemispheres, but the pressures on the horizontal faces press them together. Now, all the horizontal surfaces are together equal to a great circle of the sphere. Hence, the pressure to be overcome is the pressure of the atmosphere on a great circle of the sphere.

This is true, however many planes may be taken; but as their number increases, the surface of the steps approaches more and more nearly to that of the hemispheres. Hence, the force pressing together the hemispheres is the pressure of the atmosphere on a great circle; in other words, it is the weight of a column of mercury, having a great circle of the sphere for base, and the height of the barometer for height.

The surface of the mercury in a Torricellian tube, the section of which is 2 *square centimetres, is* 76 *centimetres above the surface of the mercury in the cistern, and the height of the vacuum above the mercury is* 8 *centimetres. If* 10 *cubic centimetres of air be passed into the vacuum, what will be the height of the mercury?*

Let x be the height of the mercury

then $84 - x =$ the height occupied by the air.

The volume of the air is therefore $2(84-x)$ cu. c.

Let P be the pressure of the air in the tube.

Since the product of volume and pressure remains the same (§ 51), and the original volume of the air was 10 cubic centimetres, the original pressure being the weight of 76 cubic centimetres of mercury,

$$\therefore \quad 2(84-x) \cdot P = 10 \times 76$$

$$\therefore \quad P = \frac{380}{84-x}$$

But the pressure of the air in the tube and the weight of the column of mercury balance the atmospheric pressure, which is equal to that of 76 cubic centimetres of mercury.

$$\therefore \quad P + x = 76$$

$$\text{or} \quad \frac{380}{84-x} + x = 76$$

$$\therefore \quad x = 60 \text{ nearly.}$$

To find the height of a station by means of a barometer, supposing the temperature and force of gravity to be constant.

Suppose AZ to be a column of the atmosphere resting on a base A, whose area is unity, and suppose the column to be divided into n strata of equal thickness, parallel to the horizon. Let $AZ=z$, and let t be the thickness of each stratum, then $nt=z$. Let ρ be the density of the atmosphere at the surface of the earth, and ρ_1, ρ_2, ρ_3, etc., the density at each of the successive levels. Let p, p_1, p_2, etc., in like manner, denote the atmospheric pressures at the surface and at the different levels.

The difference between p and p_1 is the weight of a column of the lowest stratum, having a square unit for its base; and, since the stratum is indefinitely thin, its density throughout may be regarded as uniform, and equal to that at the lowest point.

Now, $p-p_1=$ the weight of the lowest stratum $= 1\times t\times w$;

but $w=g\,\rho$,

and $\therefore p-p_1=1\times t\times g\,\rho.$

Now, the density and pressure of an elastic fluid, at a temperature θ, are connected by the formula

$$p=k\,\rho\,(1+a\,\theta)\,;$$

Hence, if we substitute for ρ

$$p-p_1=\frac{t\,p\,g}{k\,(1+a\theta)}.$$

For the constant $\frac{g}{\kappa\,(1+a\theta)}$, write K;

$$\therefore p-p_1=t\,p\,\mathrm{K};$$

hence $$\frac{p_1}{p}=1-t\mathrm{K}.$$

Similarly, $$\frac{p_2}{p_1}=1-t\mathrm{K}.$$

....

$$\frac{p_n}{p_{n-1}}=1-t\mathrm{K}.$$

Thus, the ratio of any two consecutive terms of the series p, p_1, p_2,........p_n is the same, and therefore

the series is a geometrical progression with $n+1$ terms.

$$\therefore \frac{p_n}{p}=(1-\mathrm{K}t)^n$$

Let H be the height of the barometer at A,

" h " " at z,

then $p : p_n :: \mathrm{H} : h$

$$\therefore \frac{h}{\mathrm{H}}=\frac{p_n}{p}=(1-\mathrm{K}\,t)^n$$

As t decreases indefinitely, and consequently n increases indefinitely, the last expression is shown in algebra to become $\epsilon^{-\mathrm{K}z}$

$$\therefore \frac{h}{\mathrm{H}} = \epsilon^{-\mathrm{K}z}$$

$$\frac{\mathrm{H}}{h} = \epsilon^{\mathrm{K}z}$$

$$\text{and } \log_\epsilon \frac{\mathrm{H}}{h} = \mathrm{K}z.$$

If M be the modulus for reducing logs from base to base 10,

$$\log_\epsilon \frac{\mathrm{H}}{h}=\frac{1}{\mathrm{M}}\log_{10}\frac{\mathrm{H}}{h};$$

$$\therefore \frac{1}{\mathrm{M}}\log_{10}\frac{\mathrm{H}}{h}=\mathrm{K}z.$$

If we substitute for K its value, we have

$$\frac{1}{\mathrm{M}}\log\frac{\mathrm{H}}{h}=\frac{gz}{k(1+a\,\theta)}.$$

Hence, if h_1 be the height of the barometric column at a station whose altitude is z_1, and h_2 that at another station whose altitude is z_2,

$$\log \frac{h}{h_1} = \frac{M g z_1}{k(1+a\theta)},$$

and $$\log \frac{h}{h_2} = \frac{M g z_2}{k(1+a\theta)};$$

$\therefore$ $$\log \frac{h_2}{h_1} = \frac{g M (z_1 - z_2)}{k(1+a\theta)},$$

or, $$z_1 - z_2 = \frac{k(1+a\theta)}{g M}\{\log h_2 - \log h_1\}.$$

This formula is an approximation only to the altitude of the station, since we have considered the temperature and the force of gravity constant. The effect of the variation in the force of gravity is so trifling, that it may be safely neglected.

The Correction for Temperature.

The temperature always decreases as we ascend from the surface of the earth. But as we are ignorant of the law of this change, and as the correction is always small, we may approximate to the true value of z by taking θ a mean between the values at the two stations, and considering it constant. Let t, t' be the temperatures at the two stations, then the mean value $\frac{1}{2}(t+t')$ must be substituted for θ.

The Correction for Moisture.

The value of a is $\frac{1}{273}$, when the composition of the air is uniform, whether it be perfectly dry, or whether it contain a certain proportion of vapour. But when the temperature increases, there is generally a greater

quantity of moisture in the atmosphere; and, since the density of vapour is to that of air (the barometer standing at 76 C.) as 10 to 16, it follows that the air will become rarer on this account, or it will expand in a higher ratio than $\frac{1}{273}$ to 1°. Laplace, therefore, proposes that a should be increased to $\frac{4}{10}$ of $\frac{1}{100}$ ($=\cdot 004$) in the centigrade thermometer.

The Constant $\frac{k}{Mg}$

From several observations of Raymond, Laplace has determined that, in the latitude of 45°, this coefficient is 18336 metres; but, if the force of gravity be supposed to be constant in the column AZ, Poisson finds that 18393 metres will agree better with these observations. Hence, introducing these several corrections into the value of z, given in the last proposition, we get

$$z_1-z_2=18393\left(1+\frac{t+t'}{500}\right)(\log h_2-\log h_1).$$

Example.—Find the height of Mount Etna above the level of the sea from the following data:

	Height of Barometer.	Thermometer.
Level of the sea . .	76·8 centimetres.	22·5°
Top of Mount Etna .	51·7 "	4·5°

$$
\begin{array}{ll}
\log h_2 = \log 76\cdot8 & = 1\cdot8853612 \\
\log h_1 = \log 51\cdot7 & = 1\cdot7134905 \\
\hline
 & \cdot1718707 \\
\hline
\log \cdot1718707 & = \bar{1}\cdot2352015 \\
\log\left(1+\dfrac{t+t'}{500}\right) = \log 1\cdot054 = & \cdot0228406 \\
\log 18393 & = 4\cdot2646526 \\
\hline
\log 33319 & = \underline{5\cdot5226947}
\end{array}
$$

Therefore the height of Mount Etna is 3331·9 metres. If we wish to find the height in yards, we can substitute for 18393 metres=20115 yards.

A liquid is contained in a vessel which is made to rotate uniformly about a vertical axis; to investigate the form of the surface (Fig. 59.)

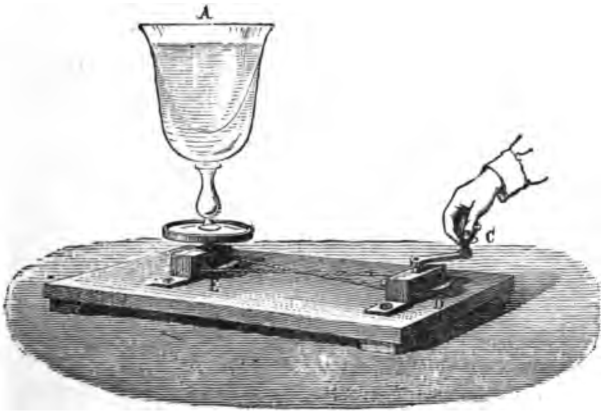

Fig. 59.

Let E A be the axis of rotation; let P be a particle in the surface of mass m; let P N be perpendicular to the axis, and P G perpendicular to the surface at P. The forces acting on P are its weight $m g$, acting downwards; the resultant pressure of the surrounding fluid, which must be along the normal P G; and the centrifugal force acting along N P. Suppose the angular velocity of the liquid to be a, then the linear velocity of P.

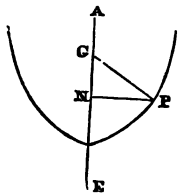

Fig. 60.

$$v = \frac{\pi r a}{180} \text{ ("Mechanics," p. 151.)}$$

but the centrifugal force

$$= \frac{m v^2}{r} \text{ ("Mechanics," p. 190.)}$$

$$= \frac{m r \pi^2 a^2}{180^2}$$

Now the sides of the triangle G N P have the directions of the forces acting on P they are therefore proportional to these forces,

$$\therefore \mathrm{N\,G} : \mathrm{P\,N} :: m g : \frac{m \pi^2 a^2}{180^2} \cdot \mathrm{P\,N}$$

$$\therefore \mathrm{N\,G} = \frac{180^2 \cdot g}{\pi^2 \cdot a^2}$$

that is, N G = a constant quantity. The line N G is termed the sub-normal, and it is proved in analytical geometry, that in the parabola the sub-normal is constant, and the parabola is the only curve which has this property. Hence the surface of the liquid

is that generated by the revolution of a parabola about its axis.

Exercises.

1. Find the force required to separate two Magdeburg hemispheres when three-fourths of the air is exhausted, the radius being 6 centimetres, the height of the barometer 76 centimetres, and the specific gravity of mercury 13·596.—*Ans.* 87·6825 kilogs.

2. A sluice gate, 10 feet square, is placed vertically in water, the upper side coinciding with the surface of the fluid; required the pressures on the upper and lower halves of the gate.—*Ans.* 7812·5 and 23437·5 lbs.

3. Required the pressures on each of the triangles formed by drawing the two diagonals.

4. A given rectangle is immersed vertically in a fluid, with one side coinciding with the surface; to draw from one of the angles to the base a straight line so that the pressures on the two parts into which the rectangle is divided may be as $m : n$.

5. If two vessels, completely filled with fluids of different specific gravities which do not mix, be placed with their surfaces in the same horizontal plane; and if a siphon, having its legs filled respectively with the two fluids, be placed in the vessels, so that each leg is in the same fluid as that with which it is filled, show that the lighter fluid will force the heavier one entirely down the leg of the siphon.

6. If the mercury in a barometer, placed in the receiver of an air-pump, stands at 30 inches, and after 12 turns sinks to 17 inches, compare the capacities of the receiver and barrel.—*Ans.* 25 : 1·21, or 20·6 : 1.*

7. A hemispherical vessel, whose weight is 12 lbs. floats upon a fluid with one-third of its axis below the surface; required the weight which must be put into it, so that it may float with two-thirds of its axis below the surface.—*Ans.* 30 lbs.†

8. A sovereign, which had the appearance, and weight in air, of genuine coin (123 grains), weighed $112\frac{1}{2}$ grains in water. Sup-

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* Log 30=1·47712; log 17=1·23045; ·9794=log 9·5377.

† The vol. of a segment of height h of a sphere of radius r is

$$\frac{\pi}{3}h^2(3r-h)$$

posing that the coin was a mixture of copper (specific gravity $= 8 \cdot 5$), and standard gold (specific gravity $= 17 \cdot 5$), what was the real value of the soverei gn ?—*Ans.* 10s. 8d.

9. In a cylindric al vessel filled with air, a cone exactly fitting it is placed with its vertex downwards, to what depth will the cone sink, supposing it to come to rest before its vertex touches the bottom of the vessel.

Let a be the height of the cylinder, b that of the cone, s the specific gravity of the cone relatively to that of mercury, and h the height of the barometric column; then the required depth is equal to

$$\frac{b\,s\,(a-b)}{h+b\,s}.$$

10. Required the pressure of sea-water on the cork in an empty bottle, supposing that its diameter is four-fifths of an inch, and that it is sunk to the depth of 600 feet.—*Ans.* 134 lbs.

11. A hollow cone without bottom rests with its base on a smooth horizontal plane. If water be poured in at the top, how high will the water rise before it raises the cone, and by that means escapes ?

12. How deep will a globe of oak sink in water, the diameter being 1 foot (specific gravity ·932) ?—*Ans.* 10·167 inches.

13. In two common pumps, each consisting of one uniform cylinder, with a valve at or near the surface of the water in the reservoir, if the greatest altitude of the piston above that surface be 20 feet in each, and the least altitude of the piston be 16 feet in one, and 17 feet in the other; find in each case the greatest height to which water can be raised when the water barometer stands at 33 feet.—*Ans.* 37ft. and 36ft.

ADDITIONAL PROBLEMS FROM THE LONDON UNIVERSITY EXAMINATIONS.

1. WHEN equal volumes of alcohol (specific gravity=0·8) and distilled water are mixed together, the volume of the mixture (after it has returned to its original temperature) is found to fall short of the sum of the volumes of its constituents by 4 per cent. Find the specific gravity of the mixture.

2. Find the pressure on a vertical rectangle, 10 inches long and 6 inches broad, immersed in water with its longer sides horizontal and with the upper one 2 inches below the surface. (One cubic foot of water weighs 1,000 ounces.)

3. A wine-bottle, which below the neck is perfectly cylindrical and has a flat bottom, is placed in pure water. It is found to float upright, with $4\frac{1}{2}$ inches immersed. The bottle is now removed from the water and put into oil, the specific gravity of which is 0·915. How much of it will be immersed in the latter fluid?

4. An inch cube of a substance of specific gravity 1·2 is immersed in a vessel containing two fluids which do not mix. The specific gravities of these fluids are 1·0 and 1·5. Find what will be the point at which the solid will rest.

5. An accurate balance is totally immersed in a vessel of water. In one scale-pan some glass (specific gravity 2·5) is being weighed, and exactly balances a one-pound weight (specific gravity 8·0), which is placed in the other scale-pan. Find the real weight of the glass.

6. At the bottom of a mine a mercurial barometer stands at 77·4 centimetres; what would be the height of an oil barometer at the same place, the specific gravity of mercury being 13·596, and that of oil 0·9?

7. Two cubic centimetres of air are measured off at atmospheric pressure. When introduced into the vacuum of a barometer they depress the mercury which previously stood at 76 cm., and occupy a volume of 15 cubic centimetres. By how much has the mercurial column been depressed?

8. A cube of brass, whose edge is 2 inches and specific gravity 8, is completely imbedded in a cube of wood, whose edge is 8 inches and specific gravity 5. Find the mean specific gravity of the whole cube.

9. The specific gravity of cast copper is 8·79, and that of copper wire is 8·88. What change of volume does a kilogramme of cast copper undergo in being drawn out into wire?

10. A mixture is made of 7 cubic centimetres of sulphuric acid (specific gravity = 1·843) and 3 cubic centimetres of distilled water, and its specific gravity when cold is found to be 1·615. Determine the contraction which has taken place.

11. The specific gravity of a mixture of two different liquids being supposed to be an arithmetic mean between those of the component liquids; required the ratio of the volumes of the latter contained in the mixture.

12. A piston, 6 square inches in area, is inserted into one side of a closed cubical vessel, measuring 10 feet each way, filled with water: the piston is pressed inwards with a force of 12 lbs. Find the increase of pressure produced on the entire surface of the vessel.

13. Two pieces of iron (specific gravity 7·7) suspended from the two scale-pans of a balance, the one in water and the other in alcohol of the specific gravity 0·85, are found to weigh exactly alike. Find the proportion between their true weights.

14. A solid, of which the volume is 1·6 cubic centimetre, weighs 3·4 grams in a fluid of specific gravity 0·85. Find the specific gravity and weight of the substance.

15. The mercury in a barometer stands at 30 inches; the section of the tube measures 1 square inch, and the vacuum above the mercury 6 cubic inches, as much air is passed up the tube as depresses the mercury to 29 inches: what would be the space occupied by the air under the atmospheric pressure?

16. A siphon barometer is so constructed that the long closed tube has an internal sectional area equal to $\frac{1}{4}$ of an inch, while the short open tube has an internal sectional area equal to $\frac{1}{2}$ an inch. Find what fall will take place in the long tube of this barometer when the true pressure of the air falls one inch.

17. In a tube of uniform bore a quantity of air is enclosed. What will be the length of this column of air under a pressure of three atmospheres, and what under a pressure of a third of an atmosphere, its length under the pressure of a single atmosphere being 12 inches?

18. A cube floats in distilled water under the pressure of the atmosphere, with four-fifths of its volume immersed and with two of its faces horizontal. When it is placed under a condenser where the pressure is that of ten atmospheres, find the alteration in the depth of immersion (the specific gravity of air at the atmospheric pressure being ·0013).

19. The contents of the receiver of an air-pump is six times that of the barrel. Find the elastic force of the air in the receiver at the end of the eighth stroke of the piston, when the atmospheric pressure is 15 lbs. to the square inch.

20. A Marriotte's tube has a uniform section of 1 square inch, and is graduated in inches, 6 cubic inches are enclosed in the shorter (closed) limb, when the mercury is at the same level in both tubes. What volume of mercury must be poured into the longer limb, in order to compress the air into 2 inches? The barometer stands at 30 inches.

21. A tumbler full of air is placed mouth downwards under water, at such a depth that the surface of the water inside it is at a depth of $25\frac{1}{2}$ feet. Compare the weight of a cubic inch of air in the tumbler with that of a cubic inch of air outside—the barometer standing at 30 inches, and the specific gravity of mercury being 13·6.

22. A certain quantity of air at atmospheric pressure has a volume of 2 cubic feet, the temperature being 55° Fahr. What does the volume of the air become when the pressure is increased by one-twentieth, the temperature meanwhile remaining the same?

23. Two liquids are mixed (1) by volume in the proportion of 1 : 4, and (2) by weight in the proportion of 4 : 1. The resulting specific gravities are 2 and 3 respectively. Find the specific gravities of the liquids.

24. A vessel in the shape of a pyramid, 5 feet high, and with

a base of 4 square feet, is filled with water. Find the pressure upon the base, and account for its being greater than the total weight in the vessel.

25. The pressure at the bottom of a well is four times that at the depth of 2 feet; what is the depth of the well if the pressure of the atmosphere is equivalent to 30 feet of water?

26. A and B are vessels full of water, with circular and horizontal bases, 12 inches and 8 inches in diameter respectively. A is 8 inches, and B is 9 inches high. Compare the pressure on the bases.

27. If a diving bell of the form of a cone be let down into the sea to the depths of 10 and 20 fathoms; find the heights to which the water will rise within it, its axis and the diameter of its base being each 10 feet, and the barometer standing at 30 inches.

28. A substance which weighs 14 lbs. in air and 12 lbs. in water, floats in mercury whose density is 13·6. What proportion of its volume will be immersed?

Answers to the above.—(1) ·937; (2) 173 ozs.; (3) $4\frac{56}{81}$ inches; (4) ·6 in. above the common surface; (5) $1\frac{11}{24}$ lbs.; (6) 11·69256 metres; (7) $10\frac{2}{13}$ centimetres; (8) 5·8; (9) 1·153 cu. cms.; (10) ·0154 cu. cms.; (11) equal; (12) 172,800 lbs.; (13) 7·7 − 1 : 7·7 − ·85; (14) 2·975 : 4·76 gms.; (15) $\frac{7}{30}$ cu. in.; (16) $\frac{2}{3}$ in.; (17) 4 in. : 3 ft.; (18) ·0024 cu. in. less; (19) 4·37036; (20) 68 ins.; (21) 7 : 4; (22) $1\frac{19}{21}$ cu. ft.; (23) 6 and 1, or 4 and $1\frac{1}{6}$; (24) 20 × 1000 ozs.; (25) 98 ft.; (26) As 2 : 1 (27) 2·806 and 3·908 ft.; (28) $\frac{35}{68}$.

PART III.

SOUND.

99. When two elastic bodies, as, for example, a hammer and an anvil, are struck together, the particles of the bodies are made to vibrate, and to impart motion to the surrounding air. The action of the vibration of the air on the organs of hearing produces the sensation of sound.

However the sonorous vibrations may be first produced, and however they may be carried to the region of the ear, they finally affect the auditory nerve by means of the air. The motion which produces sound may, however, be transmitted from one point to another by any elastic body, gas, liquid or solid. That some such body is necessary for the transmission of sound is proved by suspending a bell within the receiver of an air-pump by inelastic strings, and exhausting the receiver. As the air is withdrawn the sound diminishes in intensity, until it finally becomes inaudible.

If the bell rests on a stand or on the plate of the receiver, the vibrations of the bell are transmitted to the receiver, and then to the air.

If the receiver be filled with gas, the sound is

again heard; and if gases of different densities be used, the intensity of the sound is found to vary as the square root of the density.

100. That the particles of the air are in motion when transmitting sound, is proved by the fact that the vibrations of one body are communicated to another not in contact with it; for example, if a note be sounded on some instrument in a room containing a piano sounds in unison with it will be produced from the piano.

On the Propagation of Sound.

101. Suppose the sound to be produced by a tuning-fork, and, for the sake of simplicity, consider only one prong vibrating right and left. As it advances to the right it forces the air before it, and produces condensation of the air. The motion of the air immediately in front is imparted to that a short distance in advance, and then ceases. The air thus made to move imparts motion to the air at a still greater distance, and comes itself to rest. Thus the motion is propagated by a wave of air.

This wave produces a condensation of the air at its successive positions. Suppose now the prong of the tuning-fork to retire, the air immediately in front follows the fork, so as to fill the partial vacuum formed; then the air a short distance from it, relieved on one side of pressure, moves towards the fork, and comes to rest. The air still further away is then made to move, and so on. Here, in the

successive positions of the wave, the air is rarefied. We will call the first a condensed pulse, and the second a rarefied pulse.

Suppose now a second condensed pulse is propagated, followed by a second rarefied pulse, and the action repeated again and again, then, when the rapidity of the succession of pulses lies between certain limits, the vibration produces sound.

A condensed pulse and its adjacent rarefied pulse form a wave. When the wave is free to take any course, the intensity at any point varies inversely as the square of the distance. In air of the same density and the same temperature throughout, a pulse travels with a constant velocity, which may be determined by experiment.

Very careful experiments have shown this velocity to be nearly 1090 feet per second. Thus, at a distance of 1090 feet, the flash of a gun will be seen one second before the report is heard.

Relation between Velocity, Pressure, and Density.

102. If the pressure of the air be increased, as, for example, when it is heated without being allowed to expand, the velocity of sound is augmented. When the density is diminished, the velocity is increased. In unconfined air of constant temperature, the density and pressure increase in the same proportion; hence, the velocity of sound remains the same. Consequently, but for the difference of temperature, the velocity on the summit of a mountain would be the

same as the velocity at the base; but as the temperature is lower at the top of the mountain, the density is greater than it would be if the temperature were the same as at the base. Hence, the velocity of sound is less on the summit of a mountain than at the level of the sea.

When different gases are confined under the same pressure, the densities will be different. For example, if the density of hydrogen be taken as unity under the same conditions of temperature and pressure, the density of oxygen will be 16, that of nitrogen 14, and so on. Now, it is found that the square of the velocity varies directly as the pressure and inversely as the density, or v varies as $\frac{\sqrt{p}}{\sqrt{d}}$.

If we take the density of hydrogen as unity, the density of the air is 14·44; hence, taking the velocity of sound in air as 1090 feet per second, we can find the velocity in hydrogen thus—

$$\frac{1}{\sqrt{14 \cdot 44}} : 1 :: 1090 : x$$

$$\therefore \quad x = 1090 \times \sqrt{14 \cdot 44} = 4142 \text{ feet.}$$

The density of oxygen is 16; consequently, to find the velocity in oxygen, we have—

$$1 : \frac{1}{\sqrt{16}} : 4142 : x$$

And generally the velocity in any gas may be found by dividing 4142 the velocity in hydrogen by the

square root of the density, the pressure and temperature being supposed to remain the same.

The Motion of the Particles of Air.

103. The motion of the wave must be distinguished from the motion of the particles. If a stone be thrown into a stream, waves are seen to succeed each other from the centre of commotion to the margin of the stream; but the water does not flow in the same direction. No hollow is formed in the centre; the particles on the surface oscillate vertically, while the wave travels outwards. When a wave is propagated in air, the particles oscillate in the direction of the wave.

104. Let us suppose the wave to be produced by a vibrating piston A, at the end of a tube A B (Fig. 61).

Fig. 61.

When the piston advances to *m*, the air immediately before it is compressed, and a condensed pulse travels along the tube. When the piston retires to *n*, a rarefied pulse is transmitted, then another condensed one, and so on.

Suppose that when the point of greatest condensation of one pulse is at *p*, the next is at *q*; then the distance *p q* is the length of a wave. Between

p and q is a point l of least condensation. As the density at p and q will be greater, and the density at l less, than that of still air, there must be points, r and s, between l and q or p, where the density is a mean or equal to that of still air. Hence a wave $p\ q$, bounded by points of greatest condensation, may be divided into four equal parts by the line l of greatest rarefaction, and the lines r and s of mean density.

Qualities of Sound.

105. A series of waves following in close succession at intervals which lie within certain limits, produce a continued sound. If the waves proceed at irregular intervals, the result is *noise;* but if at regular intervals, then a *musical note* is produced.

A sound has three qualities:—

1. *Intensity*. By intensity is meant the degree of loudness of the note; it depends on the extent of the vibrations of the particles of air. The difference between the sounds produced by a cannon and a rifle is one of intensity.

2. *Pitch* is the term applied to the distinction between notes high and low, grave and acute. It depends on the length of the wave which constitutes the sound, and varies with the wave length. Sounds of different pitch travel with the same velocity. This law is evident from the fact, that when a band is heard from a distance, none of the notes are lost.

3. The *Timbre* of a note is a distinction depending on the nature of the instrument which produces the

note. The difference between notes of the same pitch from an organ and a piano is one of timbre.

The Relation of Musical Notes: the Siren.

106. Experiments have proved that notes of the same pitch are always produced by the same number of vibrations in a given time.

The number of vibrations made per second by a vibrating body may be counted in various ways. Suppose the body to be a tuning-fork, and let one prong be furnished with a small elastic steel point.

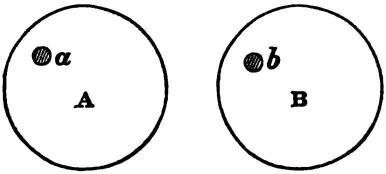


Fig. 62.

Take a piece of smoked glass, and while the fork is vibrating, draw the glass before the point, so as just to touch it without pressure. The point will mark a zig-zag, the number of angles on either side of which will give the number of vibrations of the fork during the time of transit.

A much more accurate plan of counting the number of vibrations, however, is afforded by a beautiful instrument termed a siren, the construction of which we proceed to explain.

Suppose we have two exactly equal discs A and B

(Fig. 62), perforated with circular holes, *a* and *b* of the same size and exactly the same distance from the centres. Let B be placed on A, so that their centres coincide. Let A be fixed, and let B revolve about an axis, meeting the disc at its centre. Make A the top of a conical or funnel-shaped tube, along which a current of air may be forced by means of a bellows.

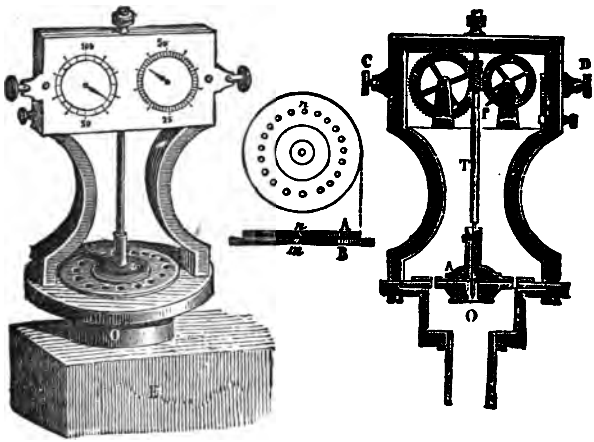


Fig. 63.

The current will not be able to pass through the aperture *a* of the disc A, unless the other aperture *b* be exactly over *a*. When B is made to revolve, once in a revolution a puff of air will escape through *a* and *b*, the current being cut off during the remainder of the revolution.

When the disc B revolves uniformly, a series of puffs will escape at regular intervals, and when the

number of revolutions per second lies between the limits of audibility, a sound will be produced.

107. The siren is an instrument constructed on this principle (Fig. 63). Instead of one hole there are usually about 25, so as to produce a note of greater intensity. All the 25 holes of the upper disc come over the 25 holes of the lower at the same instant, so that the same note is produced from each aperture, but the intensity of the whole is 25 times what it would be if all the holes in the lower disc were closed but one. The holes in the upper disc are cut obliquely, so that the air itself in passing through produces the rotation of the disc. The axis of rotation is furnished with an endless screw and toothed wheels connected with two dials. The hands and graduated circle in front of the dials give the number of puffs per second.

Suppose we wish to know the number of vibrations per second made by a body, as, for example, a vibrating string. We work the bellows of the siren and start the disc; at first no sound is heard, but when the number of vibrations exceeds 14 or 16 per second, the ear perceives a continuous but extremely low sound. If we work the bellows with greater force, the speed of the disc increases, and the pitch of the note rises in proportion, until at length we may perceive that it is in unison with the note produced by the string. By keeping up the same current of air for a definite time as, for example, a minute, we can read on the dials the number of vibrations produced in this interval, and can then calculate the vibrations per second.

The siren produces always the same sound with the same number of vibrations, whether it be played in air, in another gas, or in water.

The Musical Scale.

108. A series of notes separated from one another by certain intervals, is termed the musical scale. These intervals have long been used in music and are easily recognized by the educated ear, but by the help of the siren we are able to express the intervals by the ratios of the numbers of vibrations per second. For example, when one note is an octave above another, the number of vibrations which produce the first is double the number of those which produce the second. Let us represent by C the note produced by 512 vibrations per second, the successive octaves below C by $C_{\prime}$, $C_{\prime\prime}$, etc., and the successive octaves above by C', C'', etc.; then

$C_{\prime}$	is produced by	256	vibrations	per second.
$C_{\prime\prime}$	"	128	"	"
C'	"	1024	"	"
C''	"	2048	"	"

It is easy to calculate the wave length in each of these cases; for example, in air, sound travels with velocity 1090 feet per second, if therefore 512 vibrations are made per second, there will be 512 waves in the distance 1090 feet; consequently, each wave will be 1090÷512, or 2·13 feet nearly.

If n be the number of vibrations per second, when the first of the n waves is 1090 feet distant, the last will be at the origin; consequently the length of each

will be $1090 \div n$. Hence the length of the wave varies inversely as the number of vibrations per second, and the wave length is directly proportional to the time of one vibration.

A series of notes C, D, E, F, G, A, B, C′, having their numbers of vibrations per second in the following proportion

C	D	E	F	G	A	B	C′
1,	$\frac{9}{8}$,	$\frac{5}{4}$,	$\frac{4}{3}$,	$\frac{3}{2}$,	$\frac{5}{3}$,	$\frac{15}{8}$,	2

is termed the *diatonic musical scale.*

The wave-lengths and the times of vibration in the scale will therefore be proportional to the numbers.

C	D	E	F	G	A	B	C′
1,	$\frac{8}{9}$,	$\frac{4}{5}$,	$\frac{3}{4}$,	$\frac{2}{3}$,	$\frac{3}{5}$,	$\frac{8}{15}$,	$\frac{1}{2}$

Taking the central C as the note produced by 512 vibrations per second, the numbers of vibrations per second will be—

C	D	E	F	G	A	B	C′
512,	576,	640,	680,	768,	856,	960,	1024

Harmonics.

109. All notes whose numbers of vibrations per second are exact multiples of the number of vibrations per second of another note, are termed the harmonics of the first. If we continue the above table, and take out the harmonics, we find them to be as follows:—

Notes	C	C′	G′	C″	E″	G″
Numbers	512	1024	1536	2048	2560	3072
Ratios of numbers	1	2	3	4	5	6

The Vibrations of Strings.

110. When a cord is stretched between two points, if the centre be pulled out of its position of rest, and suddenly let go, it oscillates for some time and then returns to its first position. The distance between the extreme positions of the middle point of the cord measures the extent of the vibratory motion. The stationary points of the cord are called *nodes*, and the points of greatest motion *loops*.

The time of vibration of the same string is always the same.

111. When a cord vibrates for a certain time, the extent of the oscillations decreases, but the time of each oscillation remains the same. Hence it follows that the cord always produces the same sound. This and the other laws of the vibration of strings may be proved experimentally, by means of an instrument termed a monochord.

The Monochord.

112. The monochord consists of a long box (Fig. 64), upon which are stretched two strings; one string is fixed at one extremity, passes over a pulley, and is stretched by means of a weight P at the other extremity; the extremities of the other string

are wound round screws fixed to the box. Under the cords, upon the box, are scales of feet and inches, and aliquot parts of the length of the strings. The two cords are in the same plane, for they are

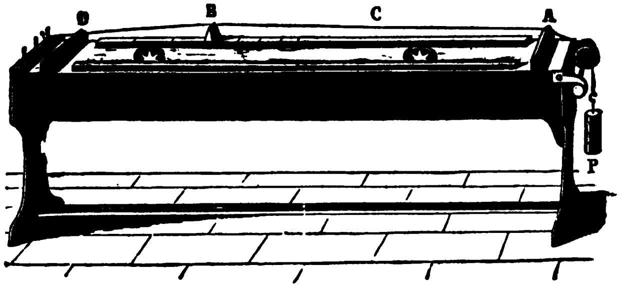


Fig. 64.

stretched across two fixed bridges of which the edges are horizontal. By placing under the cords movable bridges as B, we may increase or diminish the length of the vibrating part as we please. By this apparatus all the laws of the vibration of strings may be illustrated.

Law of variation depending on the Length.

113. The numbers of vibrations in a given time of two cords of the same material, the same diameter, and stretched by the same weight, are inversely proportional to the lengths of the cords. In order to prove this law, let us stretch between the bridges two exactly similar strings, and cause them to sound in unison; their tensions will then be equal. By the aid of the movable bridge, we divide one of the cords

into two equal parts, and make one of the strings vibrate at the same time as the other. The note produced will be the octave above. If, instead of taking half the cord, we make a third part vibrate, we shall have the fifth. We can, by similar divisions, obtain all the sounds of the scale, and if we give to the cord in succession the lengths which produce all the different sounds of the scale, we shall find that the lengths may be represented by the following numbers:—

$$1,\quad \frac{8}{9},\quad \frac{4}{5},\quad \frac{3}{4},\quad \frac{2}{3},\quad \frac{3}{5},\quad \frac{8}{15},\quad \frac{1}{2}.$$

These numbers are the reciprocals of the numbers—

$$1,\quad \frac{9}{8},\quad \frac{5}{4},\quad \frac{4}{3},\quad \frac{3}{2},\quad \frac{5}{3},\quad \frac{15}{8},\quad 2,$$

which are proportional to the numbers of vibrations in a given time required to produce the notes of the scale.

Law of variation depending on the Diameters.

114. The numbers of vibrations made by two strings of the same material and length, but of different diameters, are inversely proportional to the diameters of the cords. As before, let us stretch a string by means of weights, and then bring it into unison with one of the fixed strings. Let us now substitute for the first a string of the same material having double the diameter, and stretch it with the same weights. We shall find that it gives a note an octave lower than the fixed string. Similarly we may test the law for any other diameters.

Law of variation depending on the Tension.

115. The numbers of vibrations per second of two similar strings stretched by different weights, are found to be directly proportional to the square roots of the weights.

Law of variation depending on the Density.

116. The numbers of vibrations of two strings of the same length, thickness, and tension, are inversely proportional to the square roots of the densities of the substances composing the strings. Thus, if we take a platinum wire, and place the bridge of the fixed string so that the string vibrates in unison with the wire, and then take a steel wire of the same size, and stretch it by the same weights, moving the bridge of the string until the vibrations are again in unison, it will be found that the lengths of the fixed string are proportional to the square roots of the densities of platinum and steel; and therefore the numbers of vibrations are inversely proportional to the square roots of the densities.

These laws may be collected in a formula thus:—

Let l = the length of the string
r = the radius of a section
d = the density
T = the stretching weight
n = the number of vibrations per second

then n varies as $\frac{1}{l} \times \frac{1}{r} \times \sqrt{T} \times \frac{1}{\sqrt{d}}$

Therefore $n = a$ constant $\times \frac{1}{rl} \sqrt{\frac{T}{d}}$

The constant is so chosen that $n = \frac{1}{rl} \sqrt{\frac{T}{\pi d}}$

Superposition of Harmonics on the Fundamental Note.

117. Hitherto we have supposed each string to vibrate only as a whole, and to have nodes only at

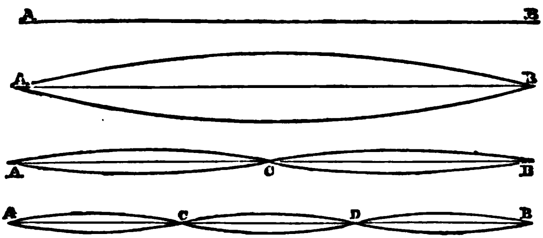


Fig. 65.

the extremities and a loop at the centre. The note produced by a string when this is the case is termed its *fundamental* note. By holding the string at its

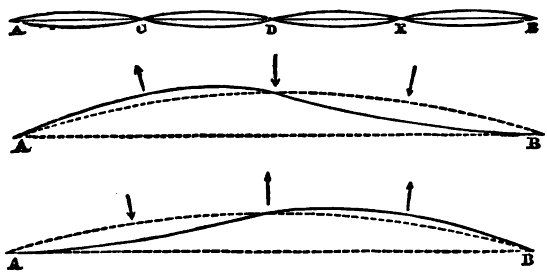


Fig. 66.

middle point, or at another point of division, and stretching one of the portions, we cause it to vibrate

in parts. We may remove the hand and the string will still vibrate in parts, and will have more than two nodes. If, for example, the string be held at the centre, there will be three nodes and two loops; if at one-third from the end, four nodes and three loops; and so on (Fig. 65).

Generally when the fundamental note is produced, one or more of its harmonics are also produced. If the vibrating body be a string, it will vibrate in halves simultaneously with the vibration as a whole. Figure 66 represents the vibration of a cord, giving at the same time the fundamental note and its first harmonic.

Sonorous Tubes.

118. When a vibrating body, as, for example, a tuning-fork, is placed over the mouth of a jar or cylindrical vessel, it is found that the intensity of the sound is greatly increased under certain conditions.

Let us take a $C_{,}$ fork and a glass jar about 15 inches long, and let us pour water gently into the jar while the fork is vibrating at the mouth; we shall find that when the depth of the air is reduced to nearly 12 inches the sound swells into a full note; when the depth is still further reduced the intensity falls again to that produced by the fork alone. The reason is that the air in a tube can be made to vibrate like any other elastic body, and that when the vibrations are suited to the length of the mass of air the motion is steady, and a continuous and definite note is produced. The vibration of the air may be caused by the vibration of a body with

suitable rapidity at the mouth of the tube, or even by blowing across the end of the tube; in the latter case the commotion produced is shaped into definite vibrations by the boundary of the column.

That it is the air in the tube which vibrates and not the substance of the tube, may be shown by taking equal tubes of different substances, as for example, tubes of wood, card-board, copper, and sounding them simultaneously; they will be in perfect unison. If sounded in succession the note will be the same.

Nodes and loops in a tube.

119. There are nodes and loops in tubes as well as in cords; that is to say, there are points where the air is motionless and where it vibrates. The existence of nodes in an organ-pipe may be shown by making one side of the pipe of glass, and letting down by means of a string a ring covered with a membrane on which some fine sand has been laid. At the nodes the sand is motionless, but at the loops it is agitated and driven off the membrane. Another curious experiment is the following: Three small gas-burners are placed on the side of the tube, two before the nodes and one before a loop. The burners are attached to small boxes, the inner faces of which are covered with thin membranes. The gas is supplied to the burners through the little boxes. When the pipe is sounded, the jets, outside the pipe at the nodes, where the sand was motionless, are agitated and extinguished, while the other remains still. This shows that at the loops although the air is in motion,

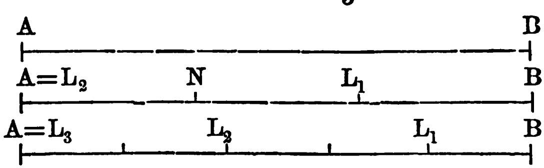
it is neither compressed nor rarefied, while at the nodes the density varies, and causes a varying pressure on the membrane.

Laws of Sonorous Tubes.

120. When the air is driven with varying force into the mouth of a sonorous tube, we find that several different notes are produced; we proceed to investigate the relation between these notes.

Notes produced by a tube closed at one end.

Let A be the open end and B the closed end. B must, of necessity, be a node; and since the density at A will be sensibly that of the air outside, A will be a loop. By § 104 the distance between a node p and a loop s nearest to it (Fig. 61) is one-fourth of the wave length, hence the longest possible wave for which the note is continuous will be four times the length of the tube. If, however, A be not the loop nearest to B, but L be the nearest and A the next, then B L must be one-fourth of the wave, and A L half; consequently the wave length will be $\frac{4}{3}$ of A B.



The lengths of the other waves which may be produced in the same tube, are—

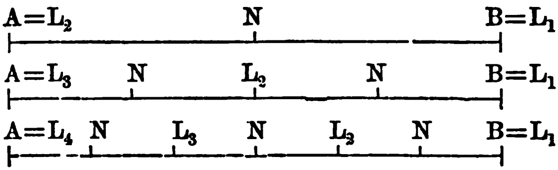
$$\frac{4l}{3},\quad \frac{4l}{5},\quad \frac{4l}{7},\quad \frac{4l}{9},\quad \ldots \quad \frac{4l}{2n+1}$$

These numbers are in the following ratios:—

$$1 : \frac{1}{3} : \frac{1}{5} : \frac{1}{7} : \ldots : \frac{1}{2n+1}$$

Notes produced by a tube open at both ends.

In this case each end is a loop; hence the column of air must be at least half a wave, since this is the distance between two consecutive loops.



If there be a loop between A and B, then the wave length will be the length A B; if there be two loops between A and B, the wave length will be $\frac{2}{3}$ A B. Hence the lengths of all the waves which may be produced with such a tube of length l are—

$$\frac{2l}{1}, \frac{2l}{2}, \frac{2l}{3}, \frac{2l}{4}, \ldots \frac{2l}{n}$$

and are proportional to the numbers

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{n}$$

From these two laws we see that the fundamental note of an open tube of given length is the same as that of a closed tube of half this length.

Interference of Vibrations.

121. In a piano there are usually two strings to a note, and it frequently happens that the strings are not exactly in unison. When this is the case, the note varies in intensity, becoming alternately louder and fainter. These alternations are termed *beats*, and arise from the interference of the two sets of waves. When the corresponding points of the two waves produced are together, the waves almost coincide, and the intensity is a maximum; but after a short interval the waves will oppose each other, and then a diminution of intensity occurs. These beats are distinctly heard by taking two tuning-forks which are in unison, and attaching a very small body, such as a fragment of wax, to one of them. When the forks are made to vibrate over a jar of proper size, the note will alternately swell to a full sound, and become too faint to be heard. When the last condition occurs, the points of greatest condensation in one wave fall upon the points of greatest rarefaction of the other, so that the waves neutralize one another.

Echoes.

122. The laws of reflection of sound are analogous to those of light. For example, the angle of incidence is always equal to the angle of reflection. Almost any kind of surface, plane or curved, rough or smooth, is capable of reflecting sound. When the reflected sound is heard distinct from the original

sound, it is termed an echo. The ear cannot perceive an interval of less than one-tenth of a second between two sounds of the same pitch, so that in order that the original sound may have a distinct echo, the reflecting surface must be at least 60 feet distant. If the distance be less, the original and the reflected sounds will be separated by less than one-tenth of a second, and will be received by the ear as a single sound.

Sound may be reflected to a focus. Thus, in an elliptical building, a feeble sound made in one focus may be heard distinctly at the other. Whispering galleries are buildings in which, from accident or design, there are surfaces which reflect sound propagated at one point to a focus at another point in the building.

EXERCISES.

1. The velocity of sound in hydrogen being 41·42 feet per second, find the velocity in each of the following gases under the same conditions of temperature and pressure:—

Air	density 14·44	*Ans.* 1090·
Oxygen	" 16·	*Ans.* 1035·5.
Carbonic acid gas	" 22·	*Ans.* 883·15.
Nitrogen	"	*Ans.* 1107·

2. How will a fall in the barometer affect the velocity of sound?

3. How does the moisture in the air affect the velocity of sound?

4. Why are sounds heard at a greater distance at night than in the day?

5. Find the wave-lengths for the following notes in air:—

C, E, G, A, C^1, G^1, C^{11}, C_1, A_{11}.

(*Note.*—Divide the velocity of sound by the number of vibrations per second.)

6. Find the wave-lengths of C, F, C^1, and A_{11} in oxygen and in carbonic acid gas.—*Ans.* C in O, 2·02 feet, and in CO_2 1·72.

7. The velocity of sound in water is 1435 metres per second; find the wave-length of E in water.—*Ans.* 2·24 metres.

8. A platinum wire 3 feet long and one-eighth of an inch thick, having a density 20·4 stretched by a weight of 5 lbs., produces a note of 240 vibrations per second, what are its first three harmonics; and what note will be produced by a similar wire 2 feet long and one-twelfth of an inch thick?—*Ans.* B_1, a note between F and G, B; a note of 540 vibrations.

9. In the last example, if a steel wire of the same dimensions, having a density of 8·5 and stretched by a weight of 6 lbs., be used, what note will be produced?—*Ans.* A note of 407 vibrations.

10. What is the fundamental note of a tube 17 inches long closed at one end, and what are the first three harmonics?—*Ans.* G^{11}, G_1, D, G.

11. What is the fundamental note of a tube 19¼ inches long open at both ends, and what are its harmonics?—*Ans.* F_1, F, G^1, F^1.

12. What must be the length of an open tube, that, when filled with hydrogen, it may produce the note C.—*Ans.* 4·045ft.

13. Two strings produce respectively 285 and 283 vibrations per second; what will be the interval between the beats?—*Ans.*

LONDON UNIVERSITY EXAMINATION PAPERS.

From January, 1862, *to January,* 1870.

MATRICULATION.

1. Explain the difficulty of opening a lock-gate when the water is at a different level within and without the lock; also, why the force required to open the gate is not proportional directly to the difference of level.

2. The weight of water is 770 times that of air; at what depth in a lake would a bubble of air be compressed to the density of water, supposing the law of Marriotte to hold good throughout for the compression?

Solution.—Let the pressure at the surface be P grammes on a square centimetre, then the pressure, P', at a depth of x centimetres is ($x \times$P) grammes. Let V be the volume of the bubble at the surface, and V' that at depth x, then by § 52 V P=V' P', but by hypothesis V=770 V', therefore by substituting for P' and V' we obtain x=769 P centimetres. If we take feet and ounces as our units x=144×·769 P feet.

3. A body weighs in air 1000 grains, in water 300 grains, and in another liquid 420 grains; what is the specific gravity of the latter liquid?—*Ans.* ·8285.

4. Sketch the common pump, describing its action, and stating the limitation to which this is subject.

5. A mercurial barometer is lowered into a vessel of water, so that the surface of the water is finally six inches above the cistern of the barometer. What *kind* of change will take place in the reading of the column of the instrument? Give a reason for your reply

6. Draw a diagram representing (in section) a single-barrel air-pump when the piston is being lowered.

Explain the action of such a pump.

If the receiver hold 64 grains of air, and 8 grains be removed by the first stroke of the piston, how much will be removed by the second stroke?—*Ans.* 7 grains.

7. If a bottle filled with air be tightly corked, and lowered into the ocean, the cork will be forced in at a certain depth. Why is this? and what will take place if the bottle be filled with water instead of air?

8. Explain why a siphon cannot be used to convey water from a lower to a higher level.

9. A solid weighs *in vacuo* 100 grains, in water ·85 grains, and in another fluid 88 grains; what is the specific gravity of this fluid?—*Ans.* ·8.

10. Explain the action of the ordinary lifting pump; find the limit of its action; and show that this will be different at the top of a mountain and at the level of the sea.

11. If a barometer were carried down in a diving-bell, what would take place? Give a rough quantitative result.

12. A solid, soluble in water but not in alcohol, weighs 346 grains in air, and 210 in alcohol; find the specific gravity of the solid, that of alcohol being 0·85.—*Ans.* 2·1625.

13. A tapering tube is bent in the middle at an acute angle, and being filled with water is inverted with the two ends completely immersed beneath the surfaces of the water in two vessels, the narrower leg being held vertical, and *its* end being only just below the surface. Point out the mode in which the water will flow; and show when the flow will cease.

14. Describe and explain the barometer; and state on what principle it is used to determine the height of a mountain.

Might it also be used to determine the depth of the sea?

15. Define clearly what is meant by specific gravity. Is there any difference in specific gravity between 4 lbs. of iron and 2 lbs. of the same metal?

A body whose specific gravity is 3·5, weighs 4 lbs. in water. What is its real weight?—*Ans.* 5·6 lbs.

16. Describe, by means of a diagram, the common pump. State the limit to its action and the cause of the same.

17. Explain the action of the siphon. Is there any limit to the height of an embankment over which water may be carried by means of a siphon from a higher to a lower level?

18. If as much additional air were forced into a closed vessel as it previously contained when in communication with the atmosphere, what would be the pressure on a square inch of the internal surface?

19. Describe the forcing-pump, and state why it cannot in general be replaced by a common pump.

20. State precisely what information is gained by knowing the specific gravity of a body.

Show how to find the specific gravity of a solid lighter than water, and not soluble in that liquid.

21. At what depth in a lake is the pressure of the water, including the atmospheric pressure, three times as great as at the depth of 10 feet, on a day when the height of the liquid column in a water-barometer is 33 feet 6 inches?—*Ans.* 97 feet.

22. A lump of beeswax, weighing 2895 grains, is stuck on to a crystal of quartz weighing 795 grains, and the whole, when suspended in water, is found to weigh 390 grains; find the specific gravity of beeswax, that of quartz being 2·65.—*Ans.* ·965.

23. A barometric tube of half an inch internal diameter is filled in the usual way, and the mercury is found to stand at the height of 30 inches. A cubic inch of air having been allowed to pass into the vacuum above the mercury, the column is found to be depressed 5 inches. What was the volume of the original vacuum?

Solution.—The volume of 5 inches of the tube $= \frac{22}{7} \times \left(\frac{1}{4}\right)^2$ $= 1$ cubic inch nearly.

Let x be the volume of the vacuum,

then $x + 1$ is the volume occupied by the air.

This air exerts a pressure $= 5$ inches of mercury.

But the air occupied 1 cubic inch when under a pressure of 30 inches of mercury. Hence, by applying the formula of page 49,

$$(x + 1)\,5 = 30$$

$$\therefore x = 5.$$

24. A bottle holds 1500 grains of water, and when filled with alcohol it weighs 1708 grains; but when empty it weighs 520 grains; what is the specific gravity of alcohol?—*Ans.* ·792.

25. Describe the construction and action of the common pump.

26. A tube closed at one end is filled with mercury, and the open end dipped below the surface of mercury in an open vessel. If the tube be inclined at an angle of 60° to the vertical, what is

the greatest length the tube can have so as to remain full of mercury, the height of the barometer being at the time 30 inches? —*Ans.* 60 inches.

27. A piece of cupric sulphate weighs 3 ozs. *in vacuo*, and 1·86 ozs. in oil of turpentine; what is the specific gravity of cupric sulphate, that of turpentine being 0·88?—*Ans.* $2\frac{6}{19}$.

28. If the height of the barometer rises from 30 inches to 30·25 inches, what is the *increase* of pressure (in ozs.) upon a square foot?—the weight of a cubic foot of water being taken to be 1000 ozs., and the specific gravity of mercury 13·56.—*Ans.* 282·5 ozs.

29. Describe an experiment which proves that the upward pressure of a fluid on any substance immersed in it, is equal to the weight of the fluid displaced by the substance. Give a sketch showing the arrangement of the apparatus.

30. If a bladder containing 300 cubic inches of air under a pressure equal to that of 30 inches of mercury be sunk 240 feet below the surface of water, the barometer standing at 28·5 inches, to what volume will the air in the bladder be compressed? [Specific gravity of mercury =13·6.]—*Ans.* 37·2 cubic inches.

31. A piece of metal weighs 211·6 grains *in vacuo*, 187·32 grains in water, and 182·37 grains in a solution of sodic chloride; find the specific gravity of the solution.—*Ans.* 1·2.

32. Describe the common barometer, and point out the principle on which its action is based.

What change in the atmospheric pressure on a square inch is indicated by a fall of one inch in the height of the barometric column? (A cubic inch of water weighs 252·7 grains, and the specific gravity of mercury is 13·6.)—*Ans.* A decrease of pressure of 3436·72 grains, or nearly half a pound on the square inch.

33. If two liquids that do not mix meet in a bent tube open at both ends, show that at rest their heights above the common surface of contact are inversely proportional to their specific gravities (§ 45).

34. The specific gravity of mercury is 13·6, and the height of the mercurial barometer is 30 inches. What is the greatest height to which water can be raised by means of the common pump?—*Ans.* 34 feet.

35. Describe and explain the construction and action of the double-barreled air-pump.

FIRST B. SC.

1. Explain what is meant by fluid pressure, and how it is measured. Show that the pressure of a fluid at rest increases with the depth.

A cylindrical vessel standing on a table contains water, and a piece of lead of given size supported by a string is dipped into the water; how will the pressure on the base be affected (1) when the vessel is full, (2) when it is not full? and in the second case, what is the amount of the change?

2. Describe the mercurial barometer.

If the mercurial barometer stand at 29 inches, and the density of mercury compared with that of water be 13·57, find what would be the height of a water-barometer.—*Ans.* 38·8 feet.

3. Give a description of an ordinary steam-engine.

What is a high-pressure steam engine?

4. Explain what is meant by the transmission of fluid pressure; and describe the construction and action of Bramah's press.

5. Investigate the conditions of equilibrium of a floating body; and describe the hydrostatic balance.

A wooden sphere has a small hole drilled in it, and is placed in water. Find its positions of equilibrium; and state which position is of stable, and which is of unstable equilibrium.

6. Describe the construction and use of a barometer. How may it be employed in determining the height of a balloon?

What would be the effect of making a small aperture in the longer branch of a barometer?

7. The water above the empty lock of a canal is 8 feet higher than the base of the floodgates, which are 4 feet broad, and provided with handles 10 feet long; find what force would have to be applied to the extremity of the handle to force open a floodgate, without previously letting in the water, assuming a cubic foot of water to weigh 1000 ozs. avoirdupois.—*Ans.* 1600 lbs.

8. Prove that a body immersed in a fluid is lighter by the weight of that amount of fluid which it displaces.

A balance is wholly immersed in water, and a body appears to weigh 1 lb., the weights against which it is balanced having the specific gravity 8·5. What will it appear to weigh when balanced against weights of the specific gravity 11·5?—*Ans.* $\frac{115}{119}$ lbs.

9. Describe generally the air-pump, and mention some of the

improvements that have been introduced in the construction of the instrument.

10. Describe an experiment in verification of the assertion, that the surface of a heavy fluid at rest is a horizontal plane; and explain the steps by which the conclusion is arrived at.

11. When a body is floating partly immersed in a liquid, what effect will a fall of the barometer have upon the body?

12. A cylinder floats in water with its axis vertical and two-thirds immersed, when the height of the barometer is 30 inches. What change will be produced when the barometer falls to 28 inches, a volume of water weighing 800 times the weight of an equal volume of air when the barometer stands at 30 inches? (See question 14, page 62.)

13. Describe the hydrostatic press, and the method of packing the pistons.

What is the mechanical advantage of this machine?

14. The specific gravity of cast copper is 8·79, and that of copper wire is 8·88. What change of volume does a kilogramme of cast copper undergo in being drawn out into wire?—*Ans.* 1·15 cubic centimetres.

15. Enunciate Boyle's law connecting the volume and tension of a given weight of air.

If the height of the barometer changes from 755 mm. to 770 mm., what is the change in the weight of a litre of air at 0° C.? (The weight of a litre of air at 0° C. and under a pressure of 760 mm. of mercury, is 1·293 grammes.)—*Ans.* ·0255 grammes.

16. Describe the construction and action of a Smeaton's single-barreled air-pump.

17. A cylindrical wooden rod of specific gravity ·72 and 1 centimetre in diameter is loaded at one end with 9·08 grammes of lead (specific gravity=11·35); how long must the rod be in order that it may just float in water at the maximum density?

18. Prove that the pressure on each of the vertical sides of a cubical vessel filled with water is equal to half the weight of the water contained in the vessel?

19. Explain fully the effect of atmospheric pressure upon the apparent weight of bodies, and show how to apply a correction for it to the results obtained by means of an accurate balance, the weight of the unit volume of air under the circumstances of the experiment being given.

20. Give an account of Watt's most important improvements in the steam-engine.

Calculate the total horse power of an engine from the following data:—

Length of stroke . . .	48 inches.
Diameter of piston . . .	40 inches.
Number of strokes per minute .	55.
Mean pressure of steam on piston,	31 lbs. per inch.
Mean back pressure from condenser,	4 lbs. „

(1-horse power=33,000 foot-pounds per minute.)—*Ans.* 452$\frac{4}{7}$.

SECOND B.A.

1. Prove that in a heavy fluid at rest, the pressure at any point varies as the depth of that point below the surface, neglecting the atmospheric pressure.

Taking account of atmospheric pressure, and taking 33 feet as the height of the water-barometer, find at what depth in a lake the pressure is twice what it is at the depth of one yard.—*Ans.* 13 yards.

2. State the conditions of equilibrium of a heavy body floating freely in a fluid.

A flat piece of iron weighing 3 lbs. floats in mercury; and if another piece of iron weighing $2\frac{5}{26}$ lbs. is placed upon it, the flat piece is just immersed; compare the specific gravities of iron and mercury.—*Ans.* 26 : 45.

3. State the relation between the pressure and density of an elastic fluid.

Describe the construction and action of any air pump, and find the pressure of the air in the receiver after a given number of strokes of the piston.

4. A piece of cork floats in a basin of water, and the basin is placed under the receiver of an air-pump. State and explain the effect of pumping out a portion of the air in the receiver.

5. If a sound travel from one point A to another B, state the direction in which the particles of air vibrate.

Describe some experiments by which it may be determined whether loud and faint sounds travel with the same or different velocities.

How is the velocity of sound affected by an increase of density in the air, the temperature remaining constant?

6. What is meant by the pitch of a note? Define the fundamental note of a pipe.

Supposing a note to be defined by the number of vibrations made in one second, state what notes may be sounded from a given pipe closed at one end by making the proper disturbances at the open end.

7. Define a fluid; and explain how fluid pressure is measured.

Describe and account for the transmission of fluid pressure; and illustrate the practical utility of this property of fluids by a description of an hydraulic press.

8. Distinguish between a gas and a liquid; and state the relation between the pressure, density, and temperature of a gas. What are the two results of experiment from which this relation is derived?

A wine-glass is inverted, and its rim just immersed in water. What would be the effect of placing a small piece of ice in the water beneath the glass?

9. Describe the construction and action of the common pump.

State whether there would be any difference in the action of a pump if it were carried to the top of a mountain, and also whether its action is in any way affected by the character of the liquid upon which it is employed.

10. Describe in its simplest form the construction and action of a double-acting steam-engine.

Explain Watt's contrivance for producing parallel motion.

11. Explain the mode of propagation of sound in air. Point out the causes which influence the intensity of sound?

12. Explain what is meant by the pitch, intensity, and *timbre* of a note?

13. What are Tartini's beats?

14. Show that the difference of the pressures at any two points of a heavy fluid is proportional to the vertical distance between the points. Compare the pressures on an equilateral triangle placed first with one side in the surface, and secondly with an angular point in the surface and the opposite side horizontal, the plane of the triangle being in both cases vertical.

15. Find the conditions that a given body may float in a given position in a given fluid.

A hollow tin cone is constructed of equal thickness throughout,

and the base is closed by a circular piece of tin of the proper size. It is then totally immersed in water. If the weight of the cone be equal to the weight of water displaced, find the forces necessary to hold the cone with the axis horizontal.

16. Explain the construction of the common pump. Explain why there is a limit to the height to which it will raise water.

17. Explain the mode of propagation of sound. What are the various causes which obstruct the propagation of sound?

18. Explain how vibrations may be excited in a tube so as to produce a musical note. What is meant by the fundamental note of a string or tube? What is the series of harmonics which can be obtained respectively from a tube stopped at both ends, at one end, and at neither end?

19. Define a fluid. Show that any pressure applied to the surface of a fluid is transmitted equally to all parts of the fluid.

20. Find the atmospheric pressure on a square inch, assuming that the height of a column of water supported by the atmospheric pressure is 30 feet, and that a cubic fathom of water weighs six tons.

21. Describe the air-pump; and find the density of the air in the receiver after a given number of strokes.

If the capacity of the receiver be ten times that of the barrel, show that after three strokes of the piston the air in the receiver will have lost nearly a quarter of its density.

22. Show how to compare the specific gravity of a solid with that of a fluid in which the solid will float.

A piece of cork weighs half an ounce *in vacuo*; a piece of metal which weighs six ounces *in vacuo*, and $3\frac{1}{2}$ ounces in water is attached to the cork; and the two are found to weigh two ounces in water. Determine the specific gravity of the cork.

23. Show that the whole pressure on any surface immersed in fluid is equal to the area into the pressure at the centre of gravity of the area.

A cylinder is filled with equal volumes of mercury and water, and is then placed *in vacuo*. Compare the whole pressure on the sides with what it would have been if the cylinder had been entirely filled with water. (The density of mercury may be taken as thirteen times that of water.)

24. A body is placed partly immersed in water in a given position. State without proof the conditions which must be satis-

fied, that the body, if left to itself, may remain in equilibrium in the given position.

Compare the depths to which a right cone must be immersed in a fluid of twice its density, that it may be in equilibrium when (1) the vertex is downwards, and (2) the base.

25. Enunciate the law which connects the pressure, density, and temperature of an elastic fluid.

Air is confined in a cylinder surmounted by a piston without weight whose area is a square foot. What weight must now be placed on the piston that the volume of air may be reduced to half its dimensions, the temperature remaining unaltered?

26. Explain how the velocity of sound is determined.

State the various circumstances which appear to influence the velocity of sound. Show how to eliminate the disturbing effect of a wind.

27. State the laws of the transverse vibrations of strings which depend on the length and the nature of the string, and on the amount of the tension.

Explain what is meant by nodal points and nodal lines.

28. Distinguish between a musical note and a noise. Define the pitch of a note. A note sounded at a station on a railway is heard in

29. A train approaching the station; does the pitch seem the same to a person in the train as to one in the station?

30. What series of notes can be sounded from a string tightly stretched between two given points A and B? Give a sketch illustrating the mode of vibration of the string in the cases of the several notes. What is the effect on the note of tightening the string?

31. Define the centre of pressure of a plane area immersed in a fluid.

32. A flood-gate is 6 feet wide and 12 feet deep. Reckoning the weight of a cubic fathom of water at 6 tons, what is the total pressure on the flood-gate when the water is level with its top; and what is the situation of the centre of pressure?

33. Give an experimental method for determining the specific gravity of a gas.

34. A cubic inch of one of two liquids weighs a grains, and of the other b grains. A body immersed in the first fluid weighs

p grains, and immersed in the second fluid weighs q grains. What is its true weight, and what is its volume?—*Ans.* $V = (p - q) \div (b - a)$, $W = (bp - aq) \div (b - a)$.

35. A quantity of air contained in a spherical vessel is transferred first into a cylindrical vessel, and then into a cubical vessel, each of which would just circumscribe the spherical vessel. Compare the total pressure produced by the air on the walls of the three vessels.—*Ans.* $5\frac{1}{3}\pi^2 : 12\pi^2 : 192$.

36. What are the reasons for believing (1) that what we call sound consists of motion of the air, and (2) that this motion is of a vibratory character? Conversely, do all undulations of the air make sound?

37. Define the fundamental note of an organ-pipe. Two organ-pipes are sounding the same note; one is closed, and the other open at the extremity opposite to that at which the note is formed. Supposing each pipe to be sounding its fundamental note, state the ratio of the lengths of the pipes.

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